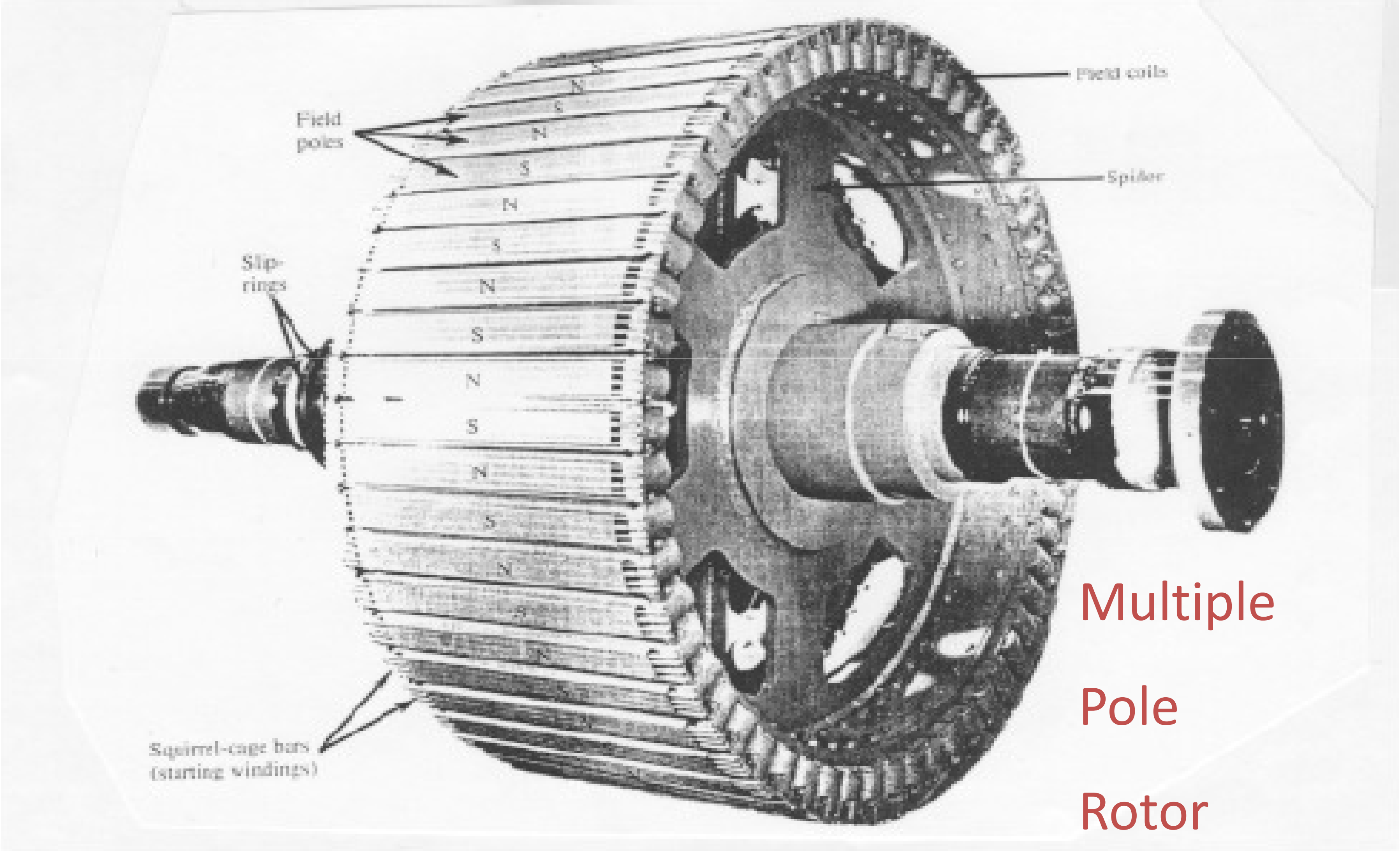
Rotating machines

Chapter 4

- *In rotating machines continuous energy conversion process takes place.*
- *Their operation depends upon utilization of Faraday's law on the electrical side, and developed torque on the mechanical side.*
- *All rotating machines consist of two parts:*
- *1- The rotor: This is the rotating part of the machine.*
- *2- The stator: This the stationary part of the machine.*

- *Both stator and rotor are concentric to each other. They are separated by small air-gap.*
- *Types of electrical machines:*
- *Electrical machines can be classified into:*
- *1- Ac machines:*
- *A- Synchronous machines.*
- *B- Induction machines.*
- *2- Dc machines.*



Multiple

Pole

Rotor

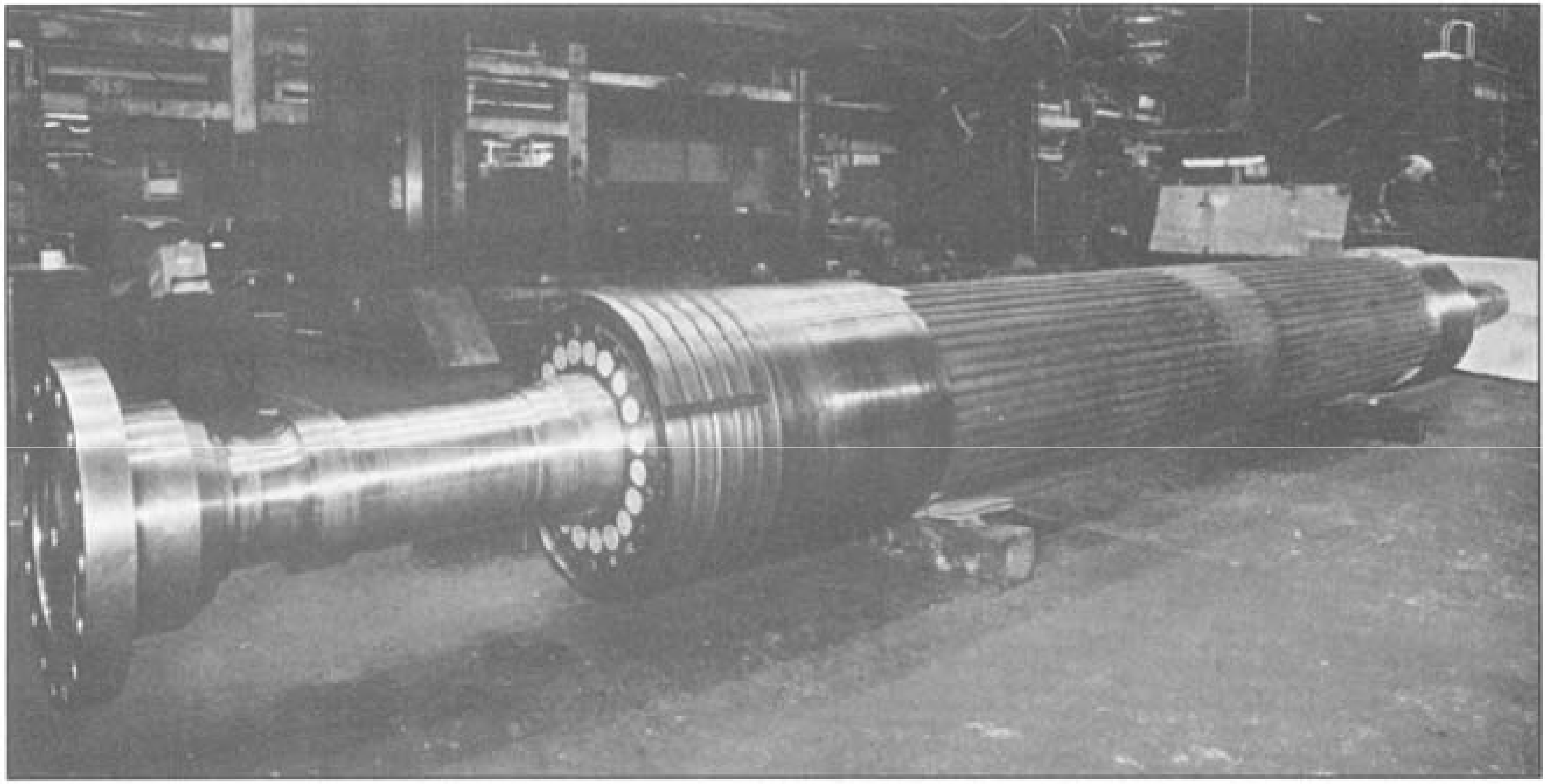
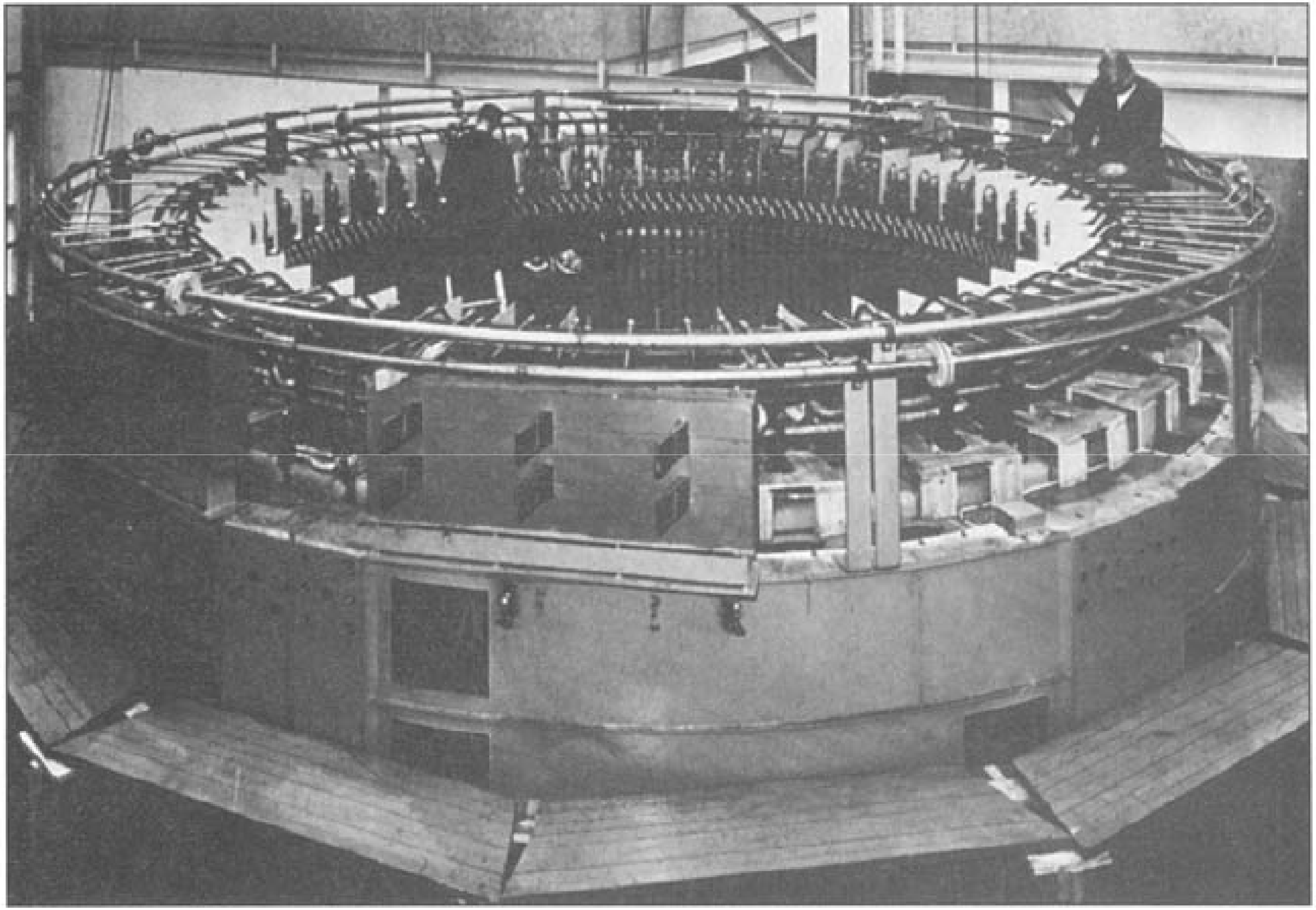


Figure 4.10 Rotor of a two-pole 3600 r/min turbine generator. (*Westinghouse Electric Corporation.*)



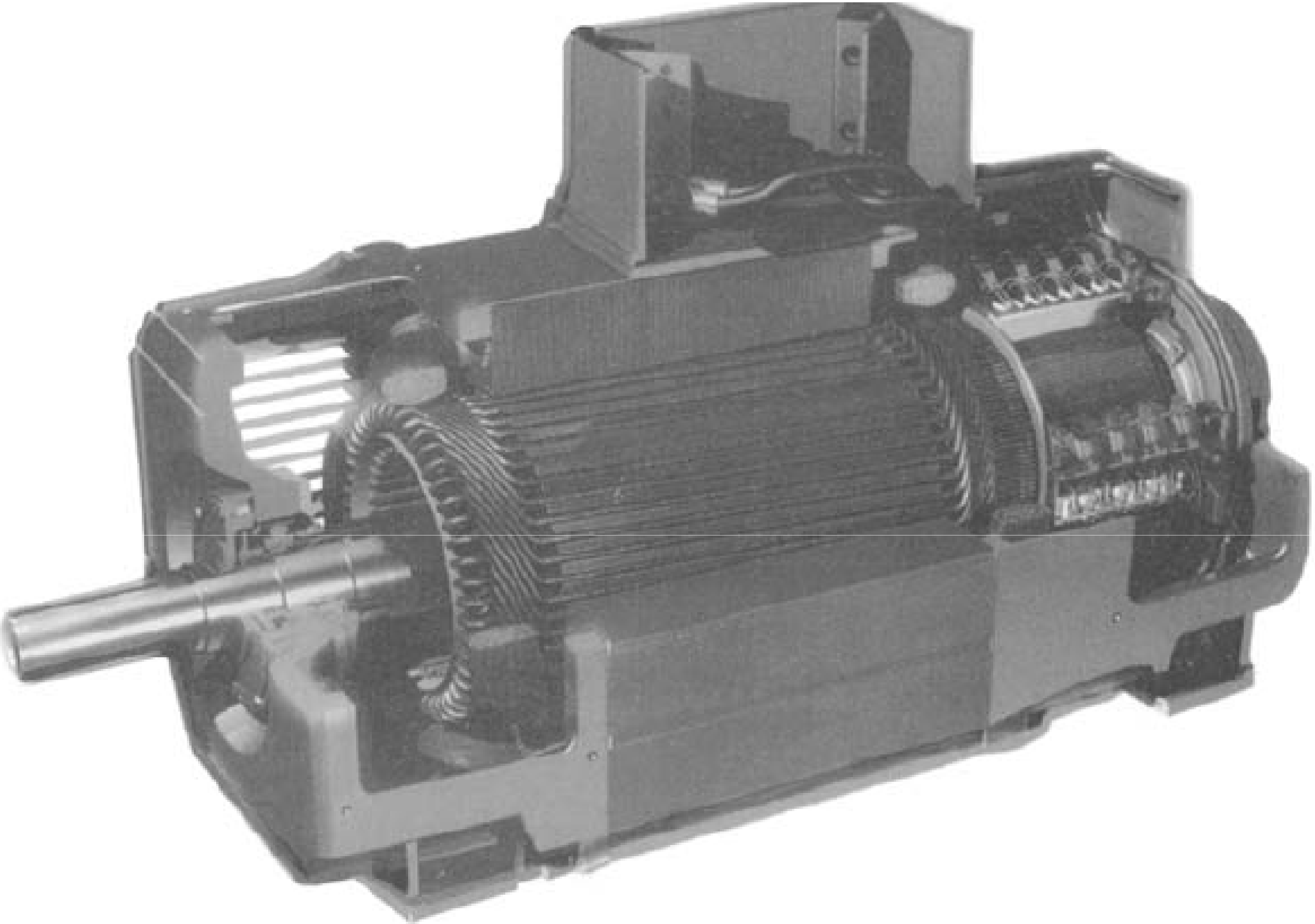


Figure 4.16 Cutaway view of a typical integral-horsepower dc motor. (*ASEA Brown Boveri.*)

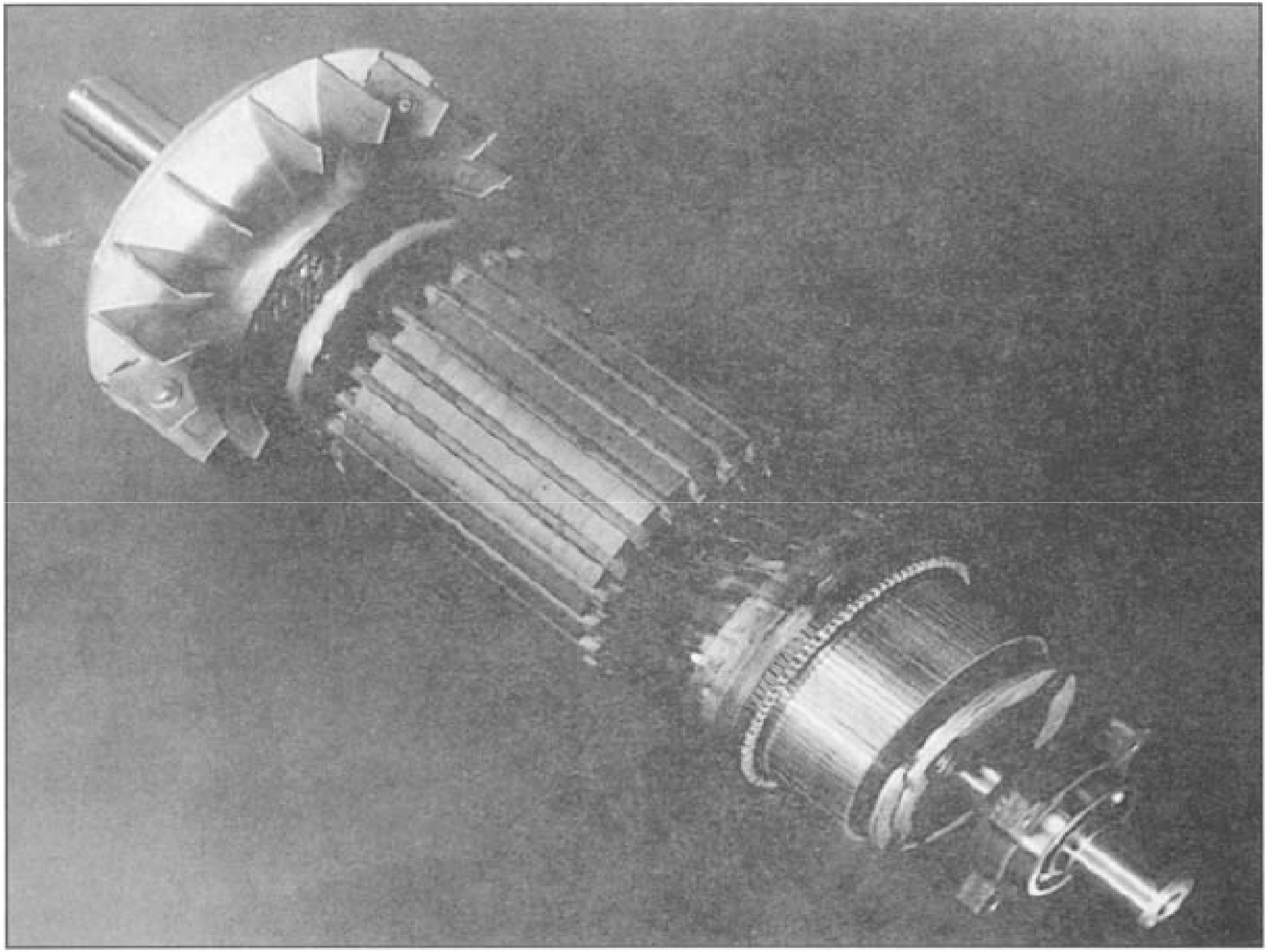


Figure 4.2 Armature of a dc motor. (*General Electric Company.*)

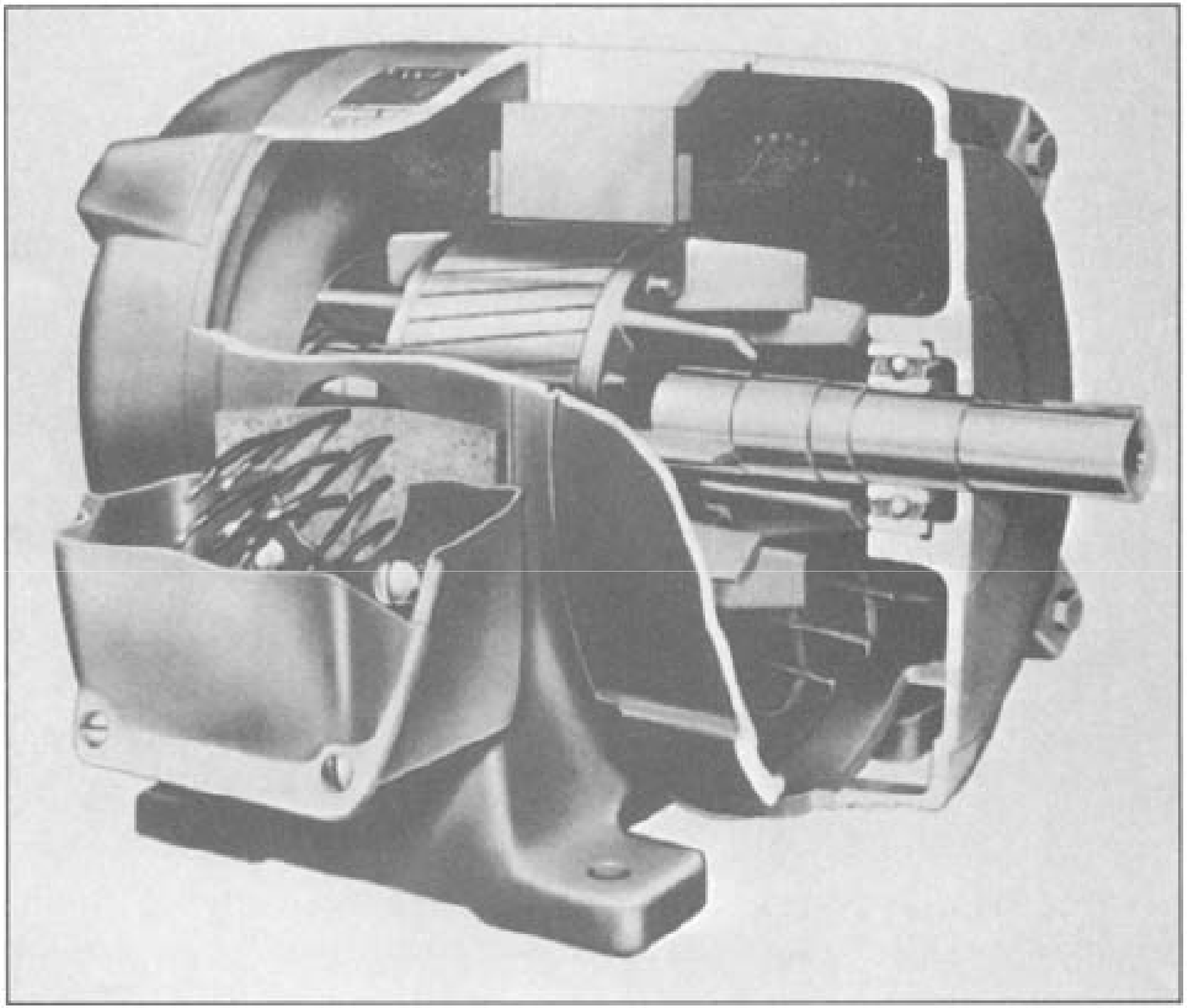


Figure 4.14 Cutaway view of a squirrel-cage induction motor. (*Westinghouse Electric Corporation.*)

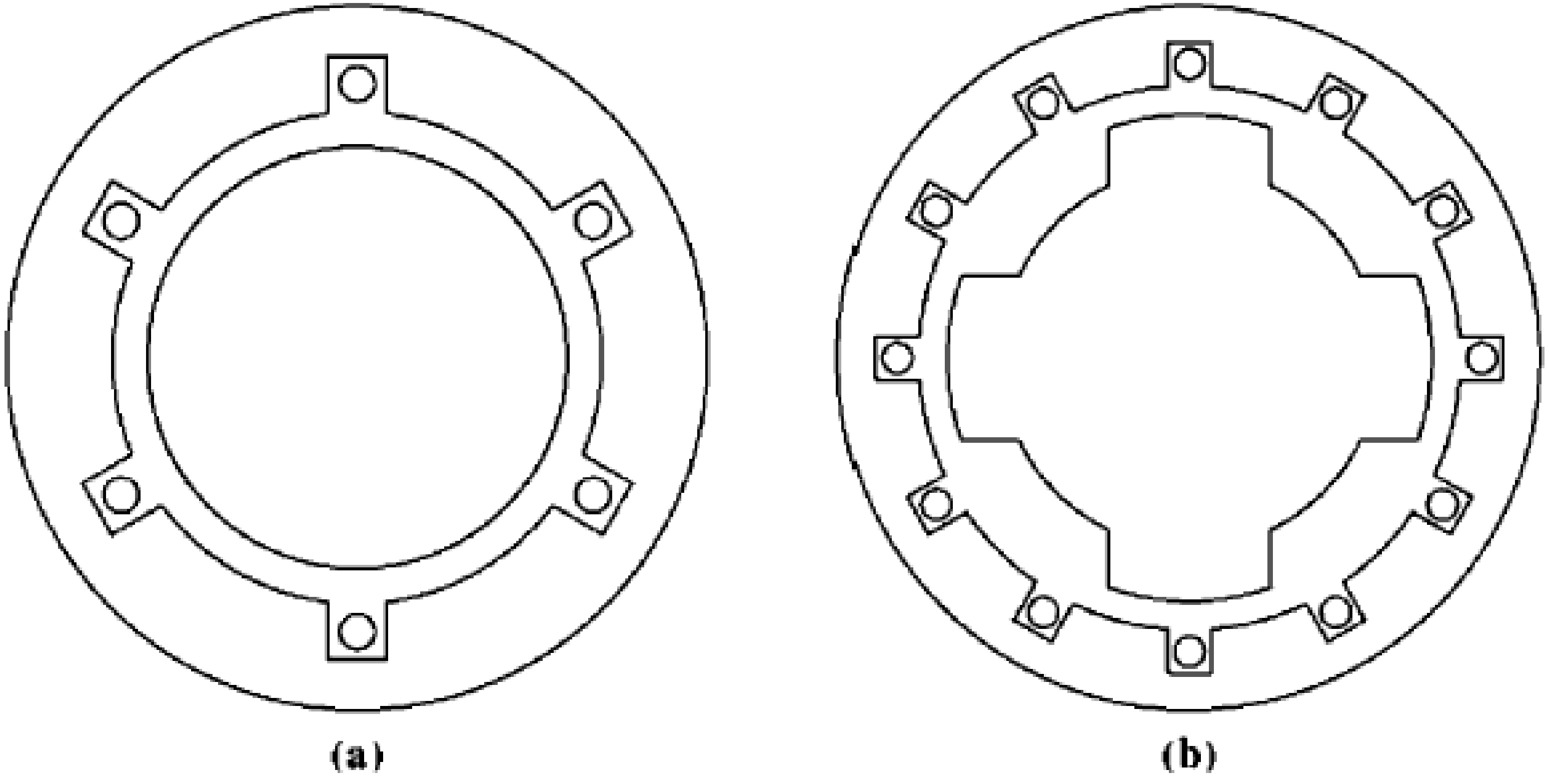
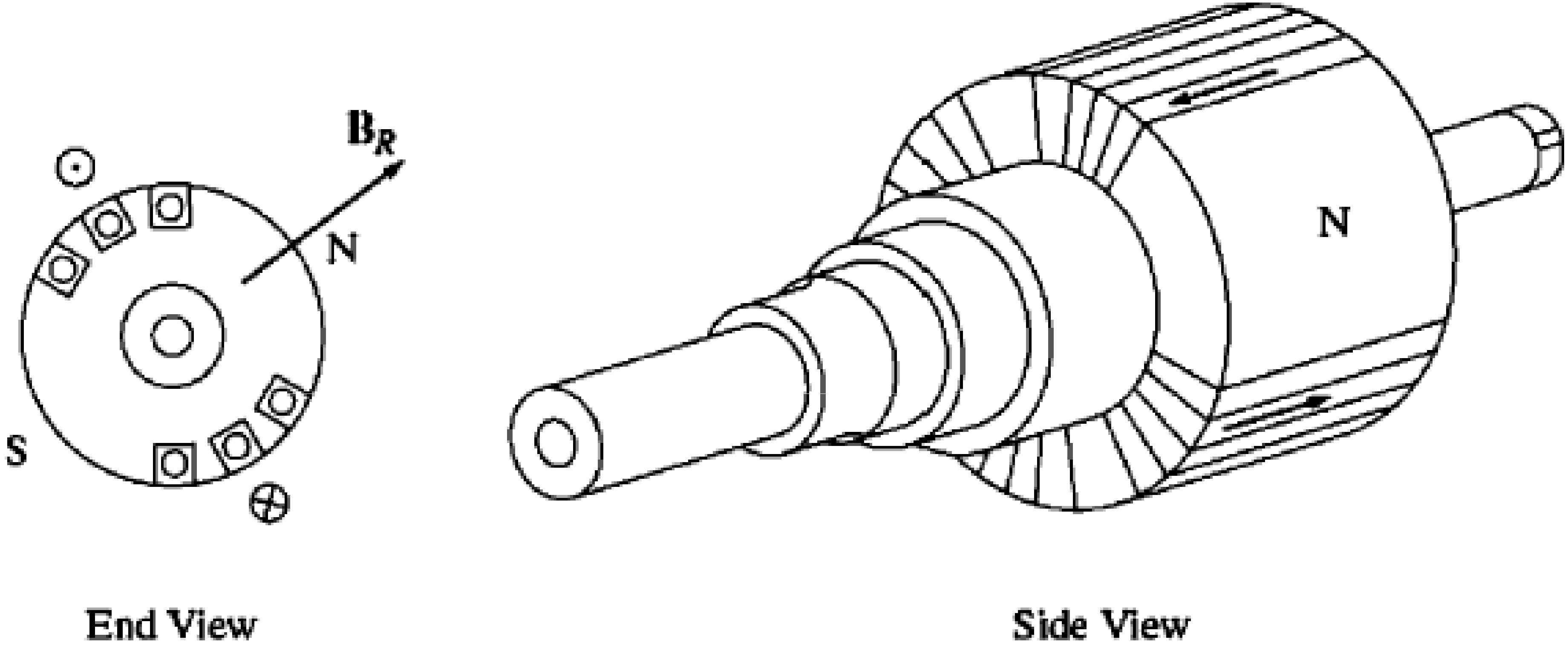


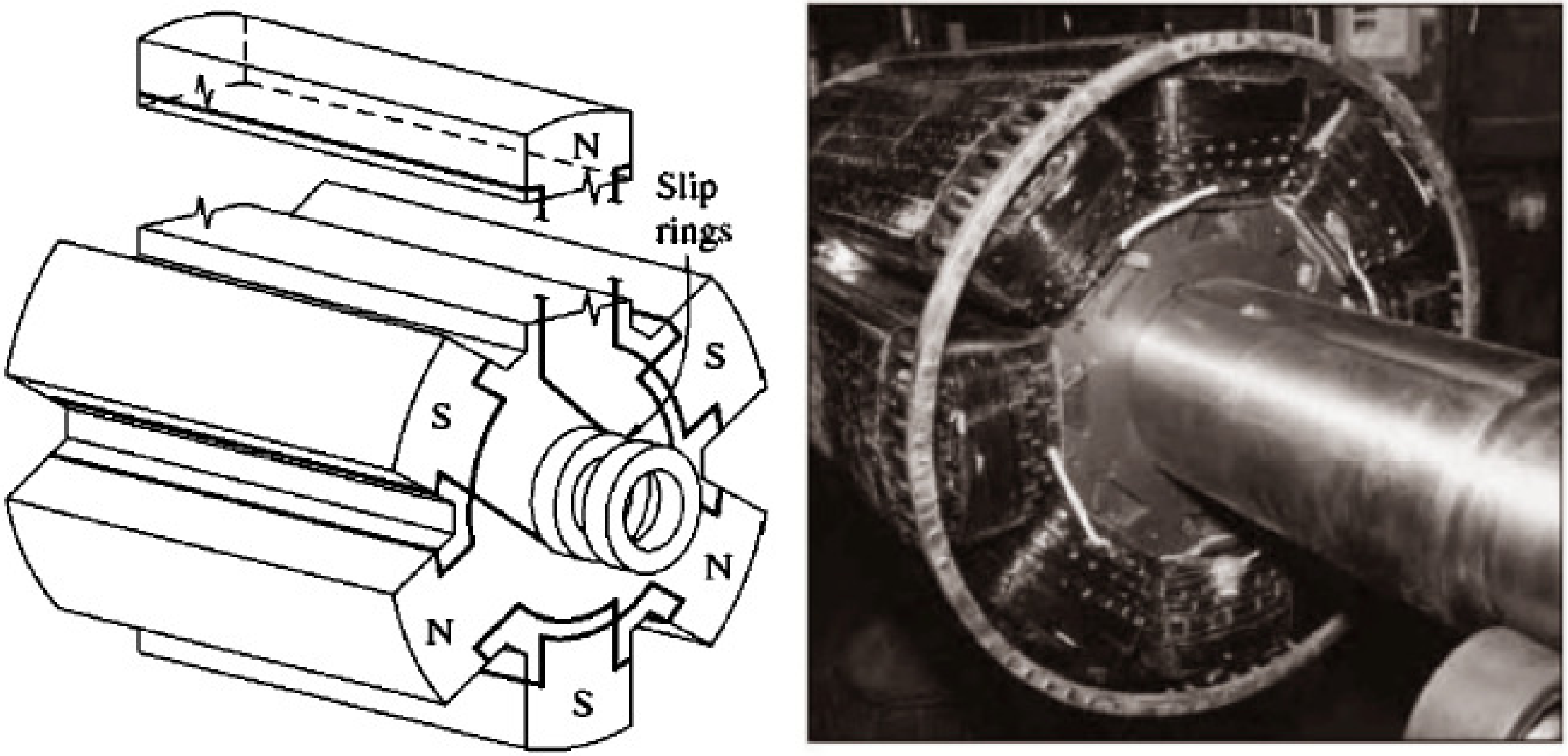
FIGURE 4–12
(a) An ac machine with a cylindrical or nonsalient-pole rotor. (b) An ac machine with a salient-pole rotor.



End View

Side View

FIGURE 5–1
A nonsalient two-pole rotor for a synchronous machine.



(a) (b)

FIGURE 5–2
(a) A salient six-pole rotor for a synchronous machine. (b) Photograph of a salient eight-pole synchronous machine rotor showing the windings on the individual rotor poles. (*Courtesy of*

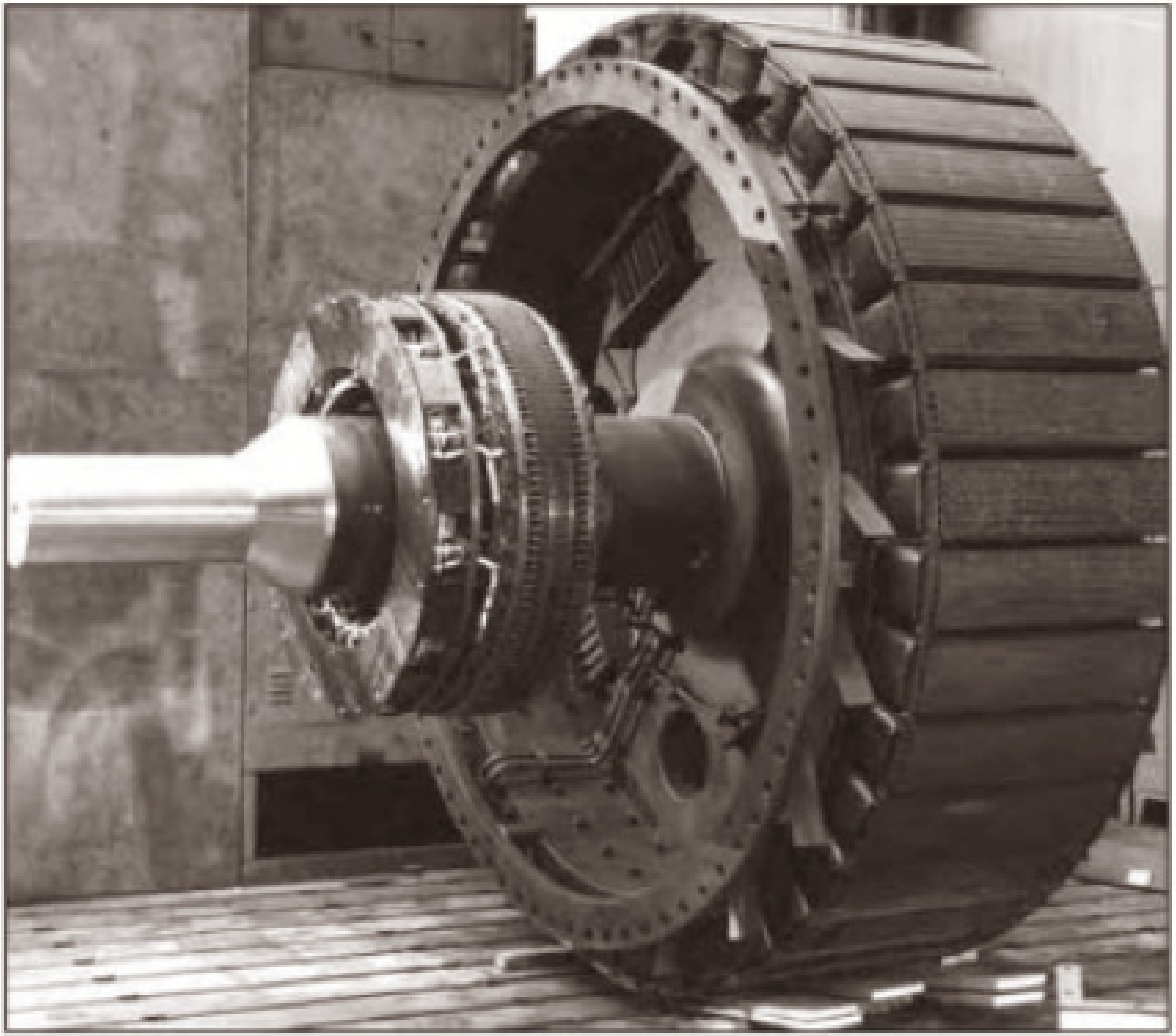


FIGURE 5–4
Photograph of a synchronous machine rotor with a brushless exciter mounted on the same shaft. Notice the rectifying electronics visible next to the armature of the exciter. (Courtesy of Westinghouse Electric Company.)

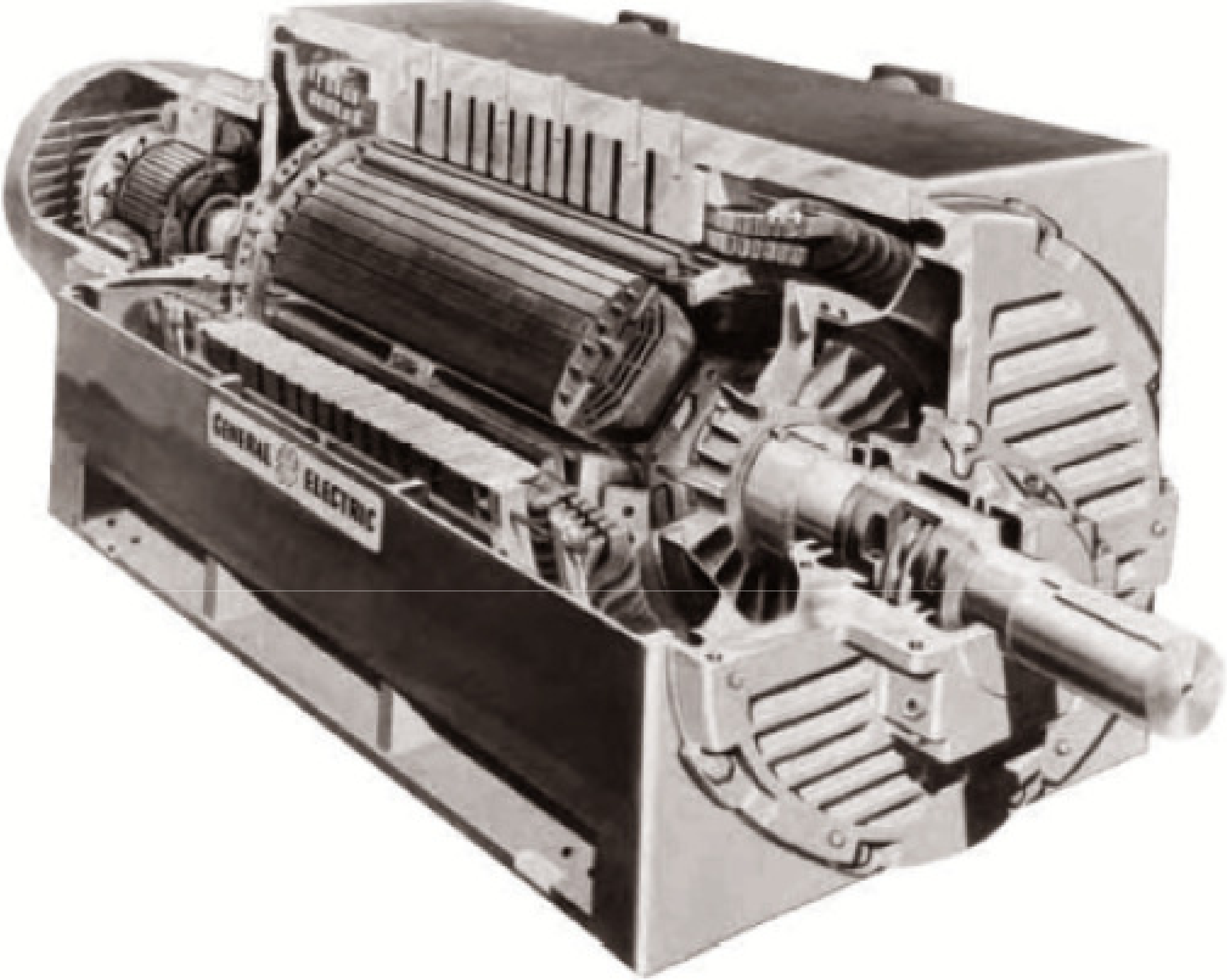
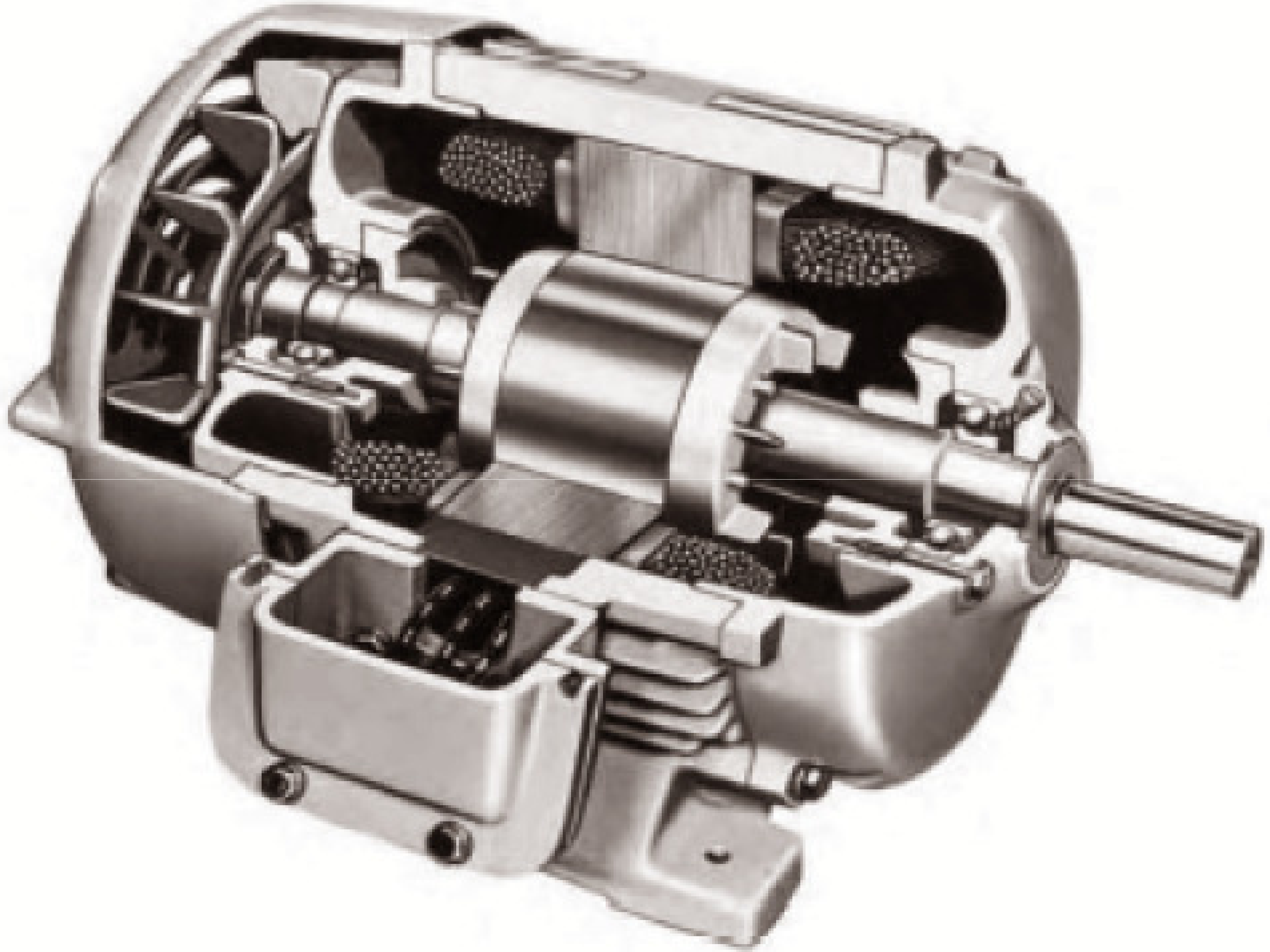


FIGURE 5–6
A cutaway diagram of a large synchronous machine. Note the salient-pole construction and the on-shaft exciter. (*Courtesy of General Electric Company.*)



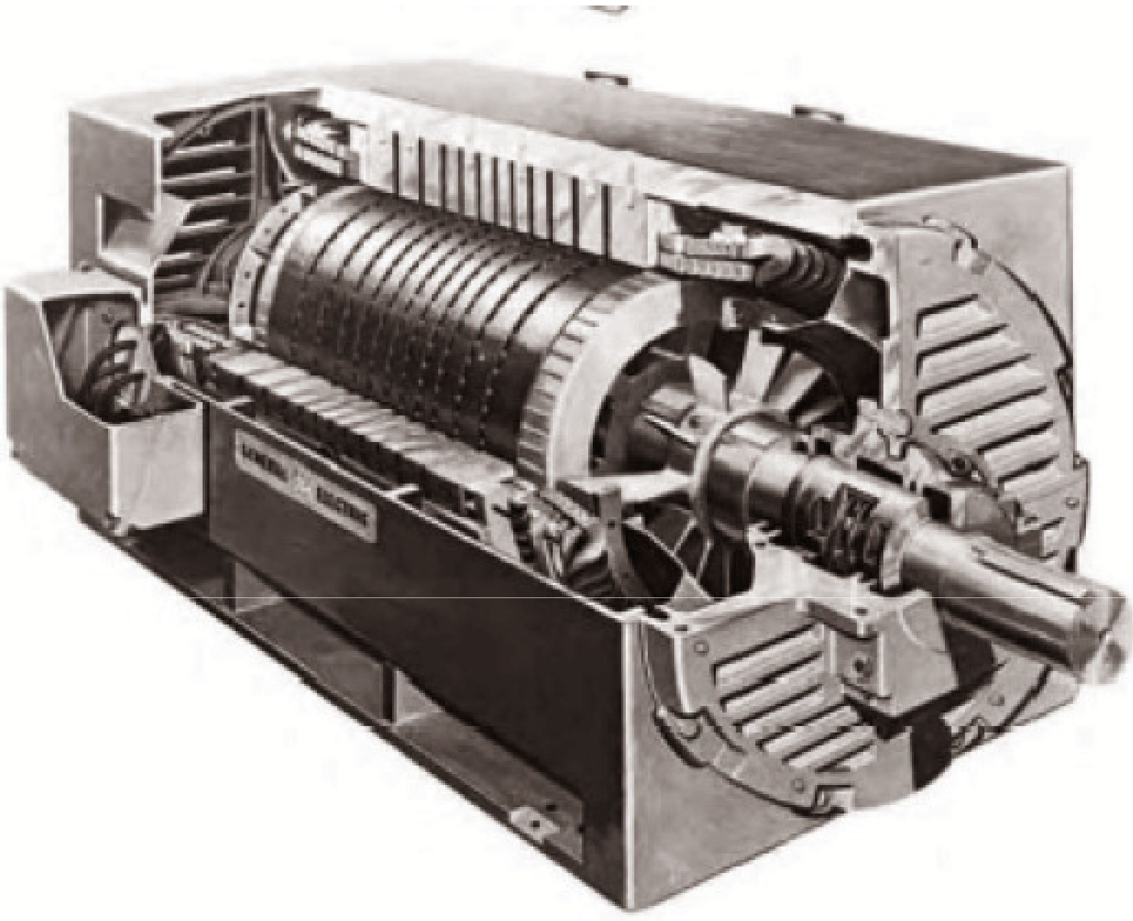


FIGURE 7–3
(a) Cutaway diagram of a typical small cage rotor induction motor. (*Courtesy of MagneTek, Inc.*)
(b) Cutaway diagram of a typical large cage rotor induction motor. (*Courtesy of General Electric Company.*)

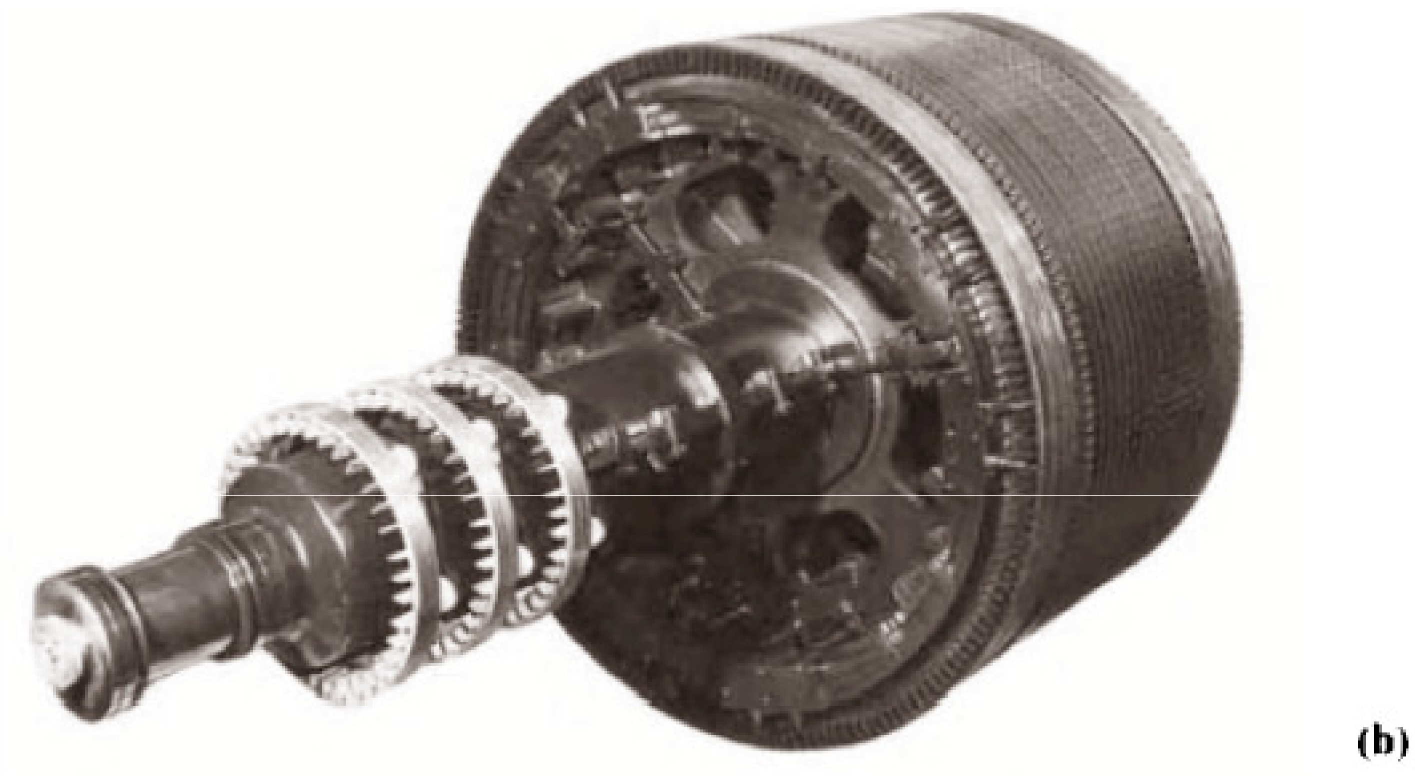


FIGURE 7–4
Typical wound rotors for induction motors. Notice the slip rings and the bars connecting the rotor windings to the slip rings. (*Courtesy of General Electric Company.*)

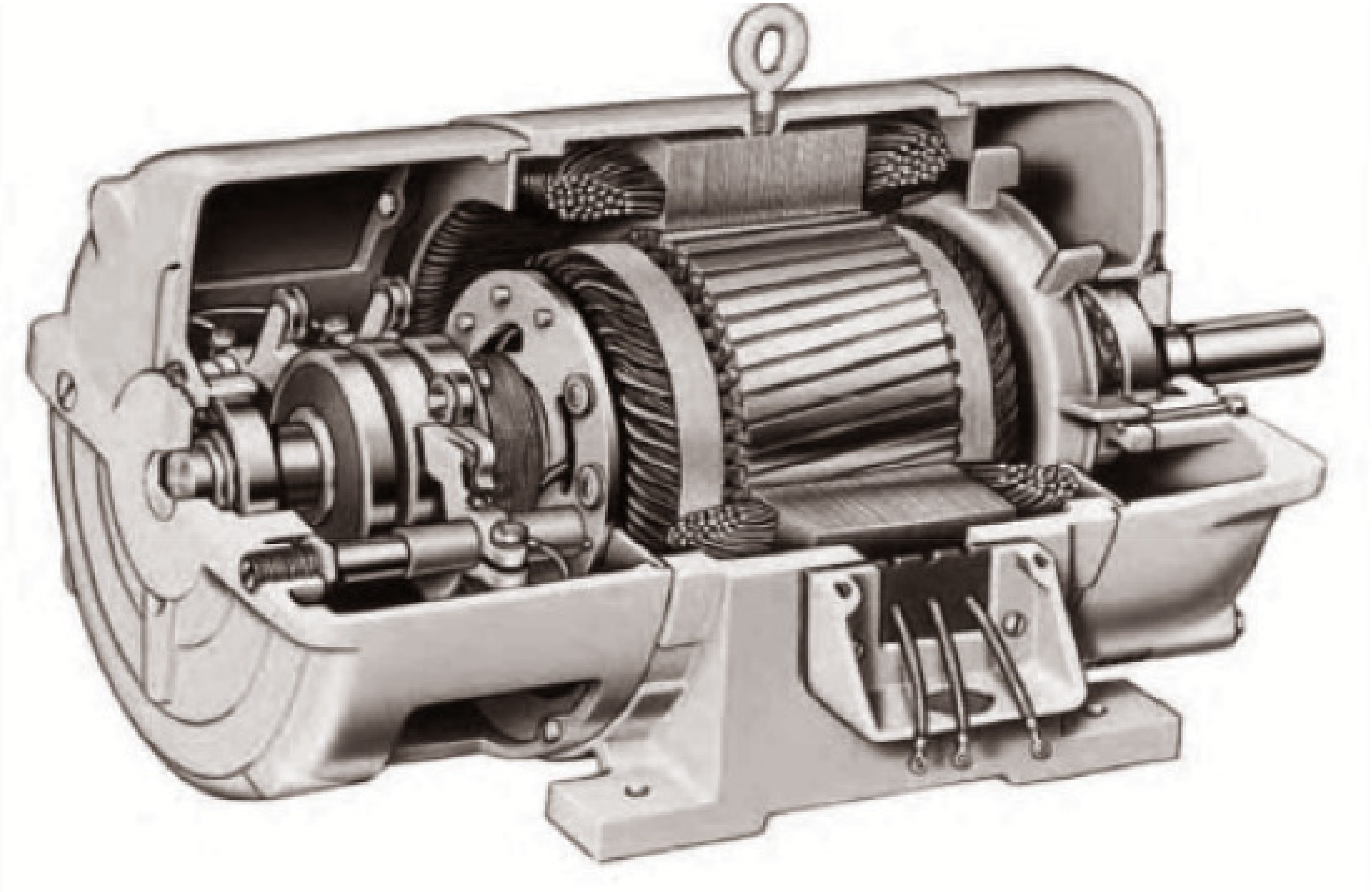
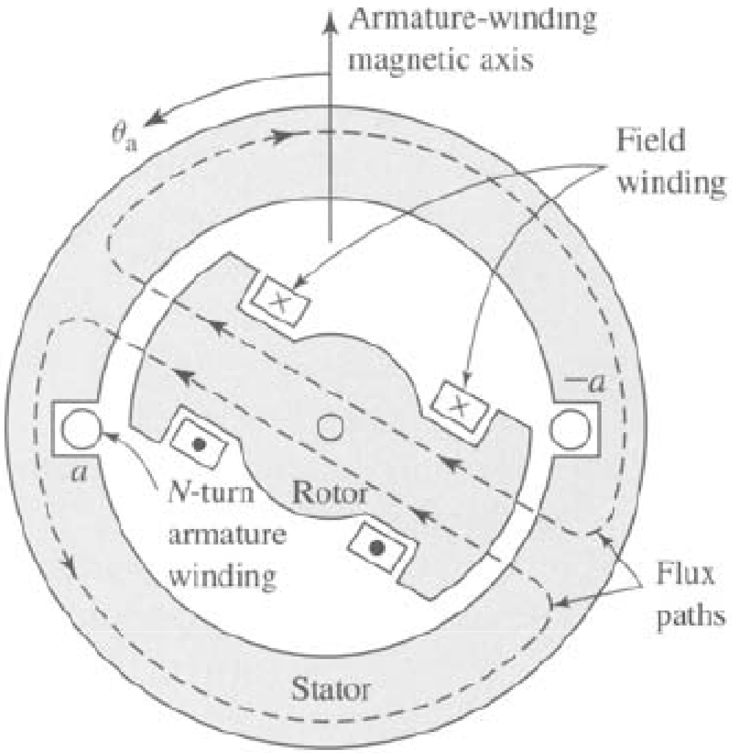
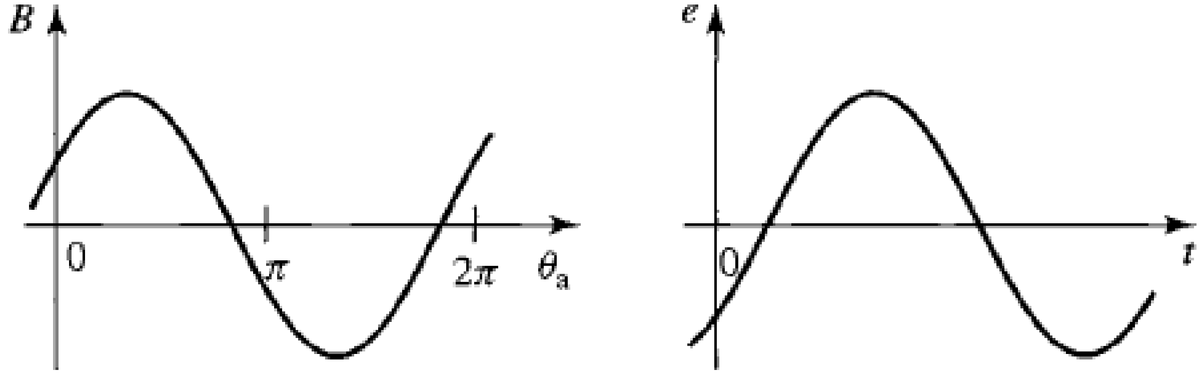


FIGURE 7–5
Cutaway diagram of a wound-rotor induction motor. Notice the brushes and slip rings. Also notice that the rotor windings are skewed to eliminate slot harmonics. (*Courtesy of MagneTek, Inc.*)

Electrical angles

- *In case of two pole machines, one complete rotation of the rotor poles, induces one complete cycle of induced emf.*
- *Then for two pole machines, the electrical angles are equal to the mechanical angles.*


Armature-winding magnetic axis
θ_a
Field winding
$-a$
a
N-turn armature winding
Rotor
Flux paths
Stator


B
0
π
2π
θ_a
e
0
t

(a)

(b)

- *In case of four pole machines, one complete rotation of the rotor poles induces two complete emf cycles.*
- *So The electrical angles are twice the mechanical angles.*
- $$\theta_e = 2\ \theta_m$$

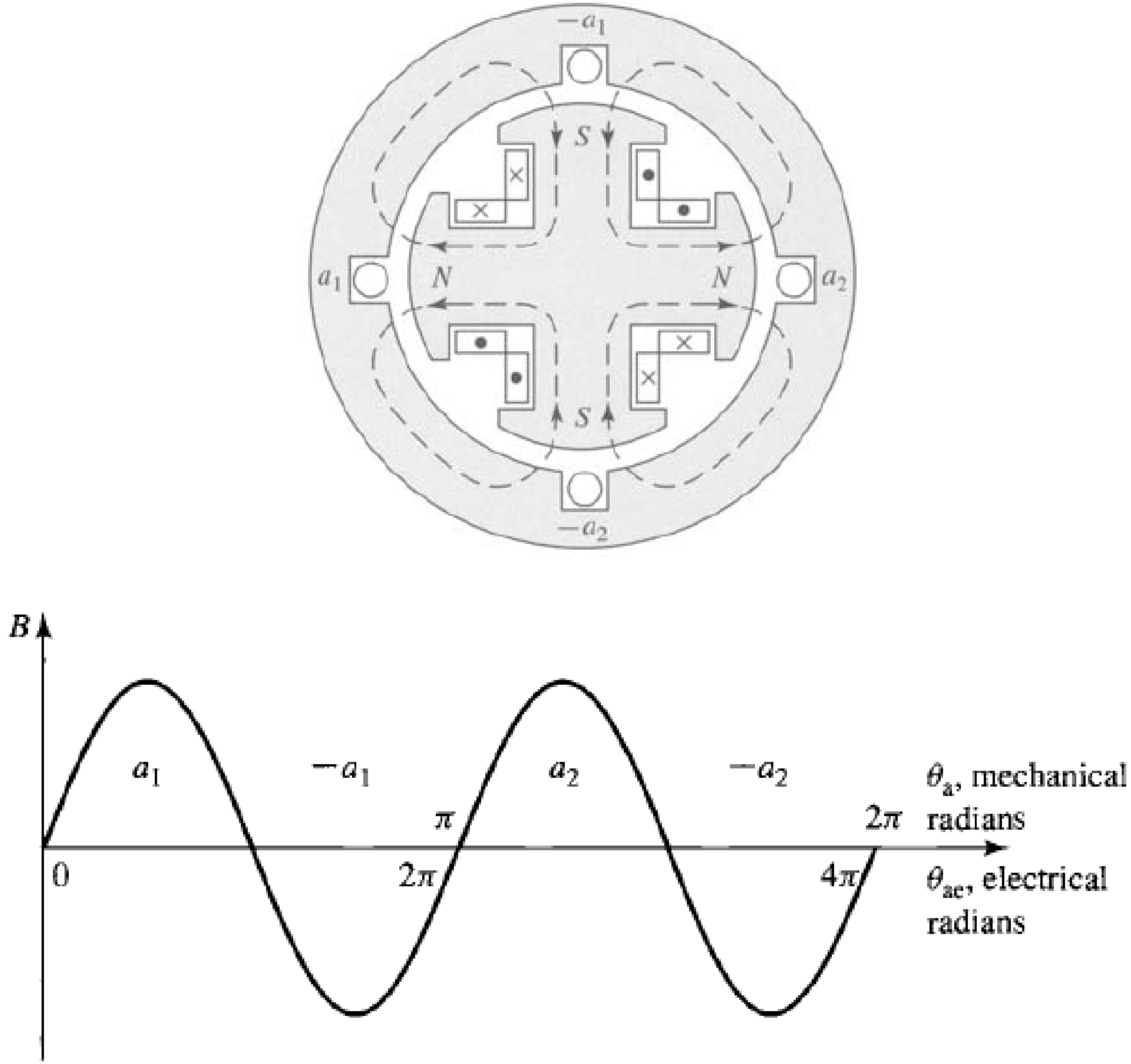


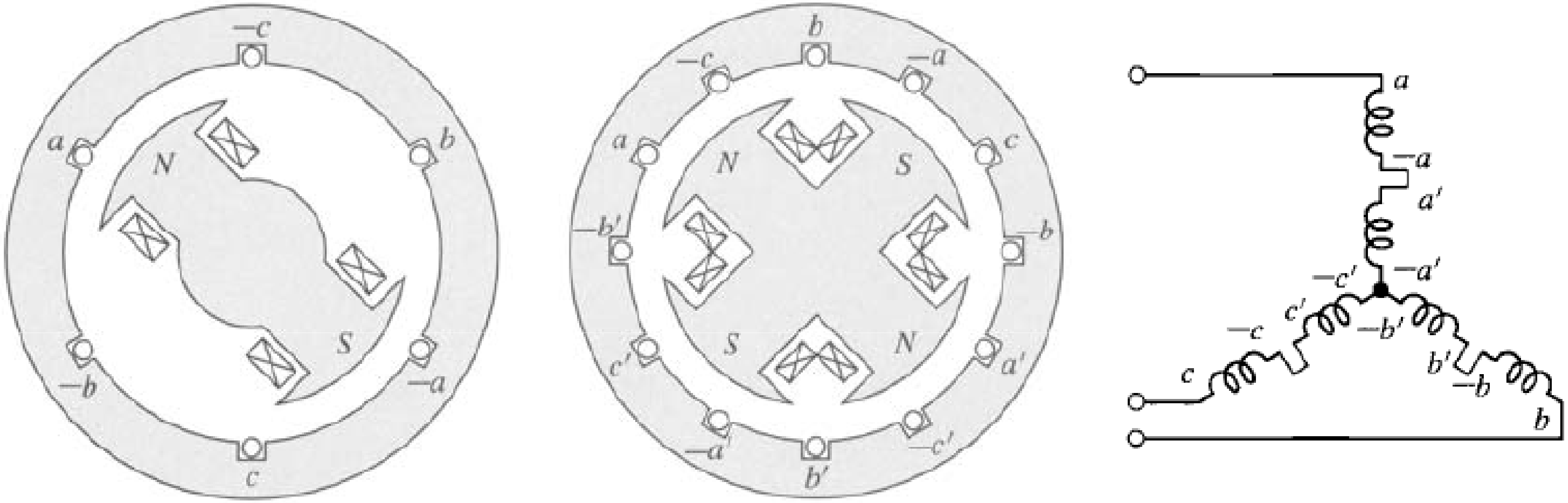
Figure 4.7 Space distribution of the air-gap flux density in a idealized, four-pole synchronous generator.

- *In general for a p-pole machine, the electrical angles are related to the mechanical angles by:*
- $$\theta_e = \frac{p}{2}\,\theta_m$$
- $$\omega_e = \frac{p}{2}\,\omega_m \quad \textit{rad/sec}$$
- $$\omega_e = 2\pi f, \qquad\qquad \omega_m = \frac{2\pi n_s}{60}$$

Three phase machines

- In three phase machine three separate windings { one for each phase } are used. These windings are space displaced by 120 electrical degrees from each other.
- Then voltages induced in these windings are displaced in phase (time) by 120 º elect.
- In case of two pole machines, only one winding for each phase are used.

- *In case of four pole machines each phase consist of two groups of windings, one group for each two poles. These groups are then connected together as required { parallel or series}.*
- *In case of p-pole machines, there will be p/2 groups of windings for each phase {one for each two poles}. These windings are then connected as required {parallel, series, combinations of parallel and series connections}.*



(a) (b) (c)

Figure 4.12 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and (c) Y connection of the windings.

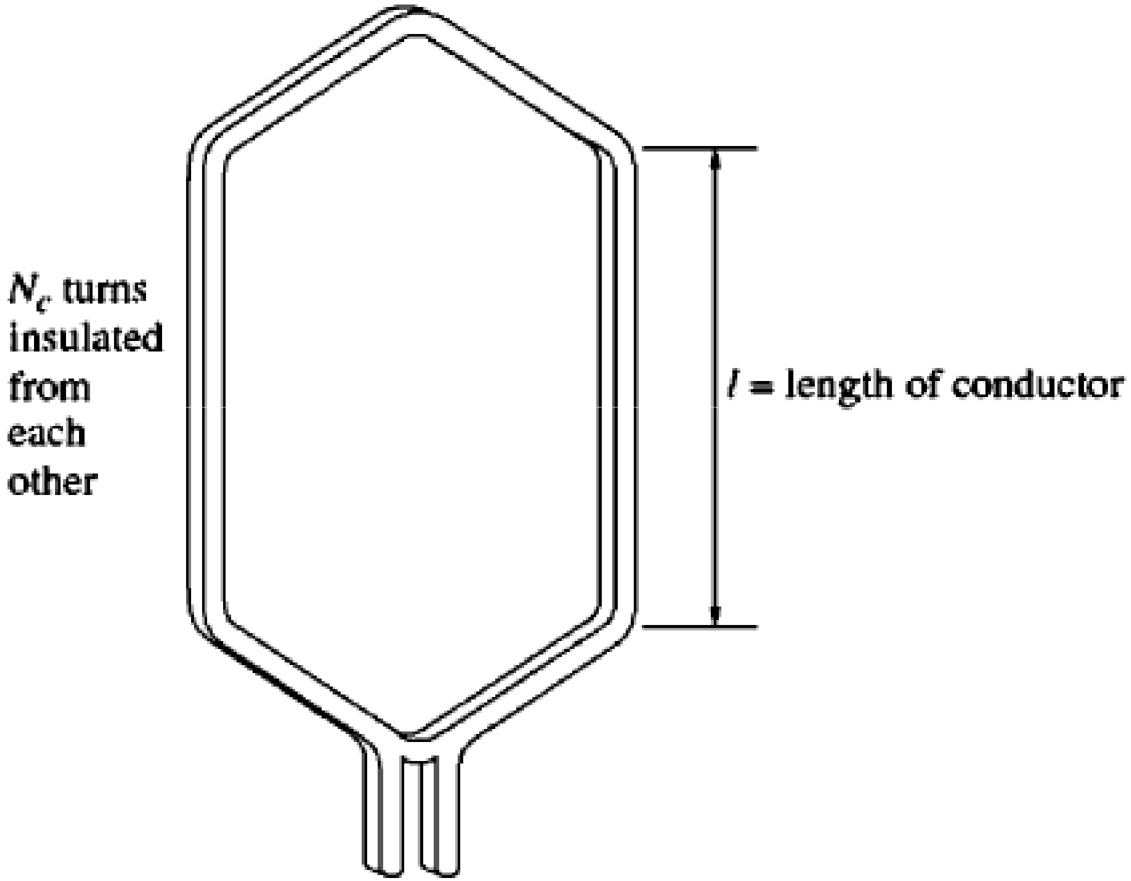
Synchronous speed

- *For a p pole machines, each one rotation produces (p/2) complete voltage cycles. So the frequency of the induced voltage is:*
- $$f = \frac{p}{2}\left(\frac{n_s}{60}\right) \quad Hz$$
- *Where* n_s *is the rotational speed in rotations per minute, and is known as the synchronous speed.*
- $$n_s = \frac{120\,f}{p} \quad r.p.m$$

- *Synchronous speed is that speed which is synchronized to the frequency of the system.*
- *This means that all generating machines must be synchronized to each other {each one rotates exactly at its own synchronous speed}.*
- *When one of these machines losses synchronism with other machines for any reason, it must be switched off and re-synchronized to the group again.*

MMF of distributed winding

- *Case 1: Single coil of N turns (undistributed):*
- *The MMF produced by this coil is:*
- $MMF = N.i \quad A.t$
- *This mmf has a rectangular space distribution as shown below.*


N_c turns insulated from each other
l = length of conductor

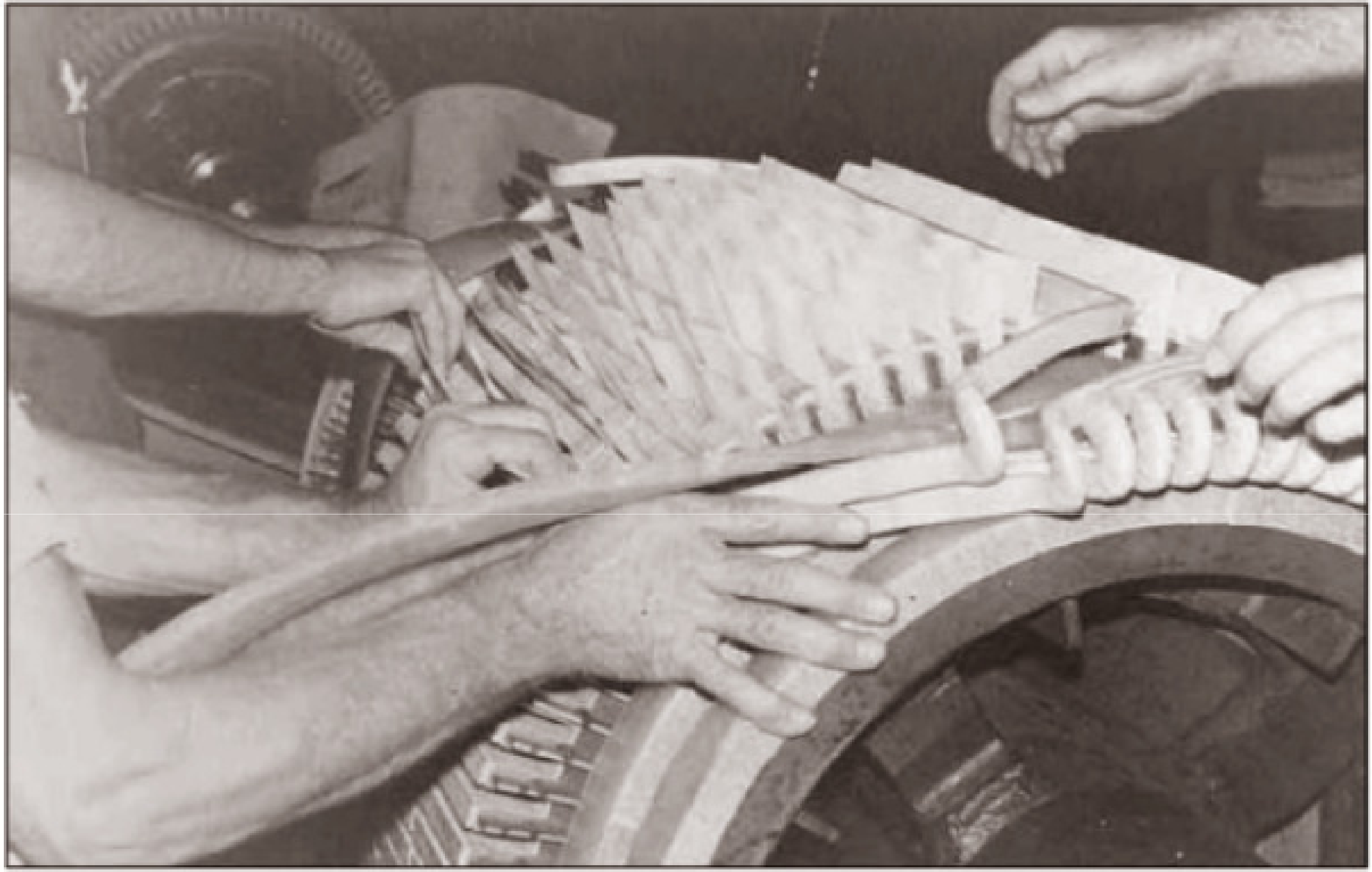
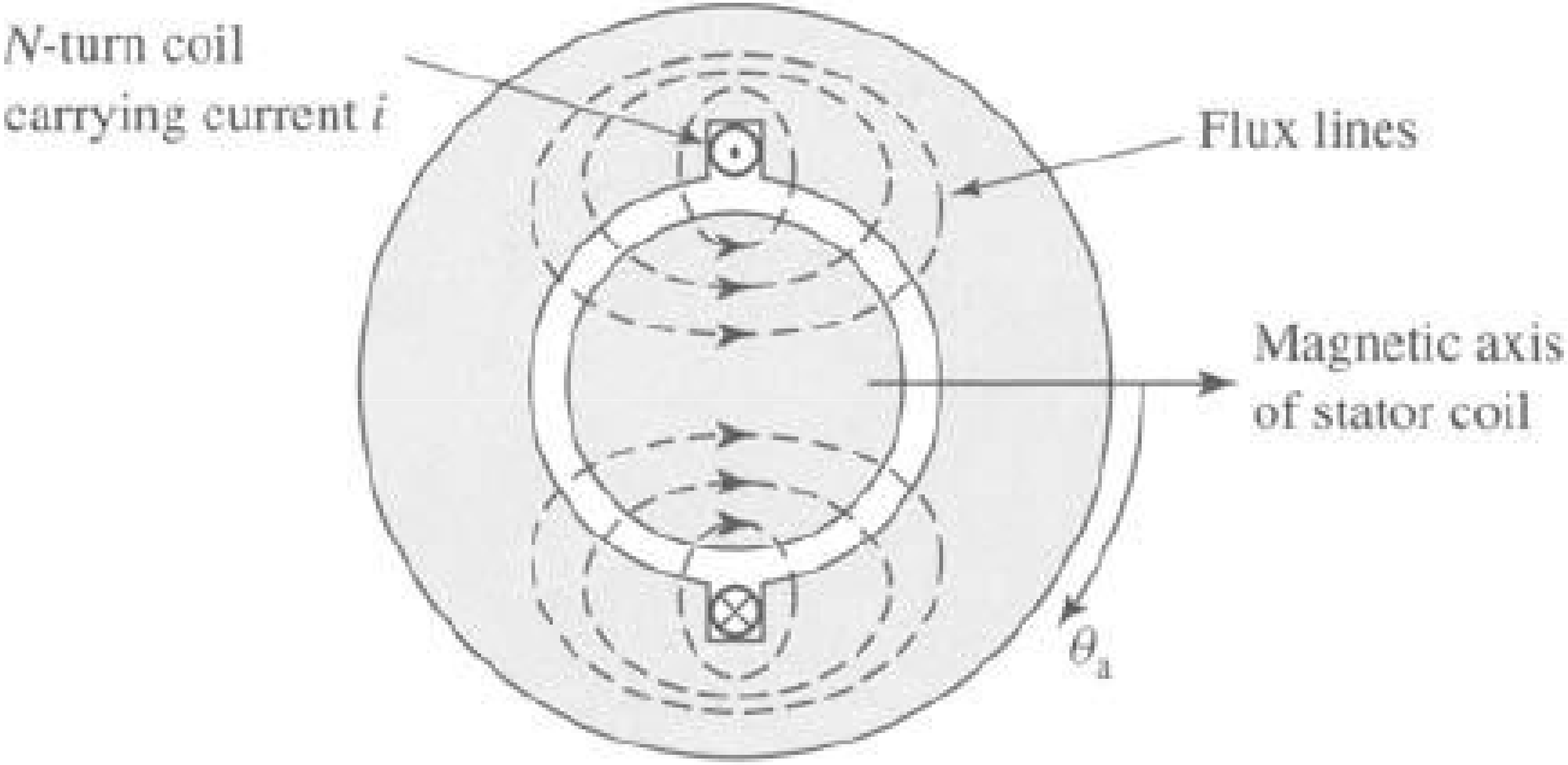


FIGURE 8–12
The installation of preformed rotor coils on a dc machine rotor. (*Courtesy of Westinghouse Electric Company.*)


N-turn coil
carrying current i
Flux lines
Magnetic axis
of stator coil
θ_a

(a)

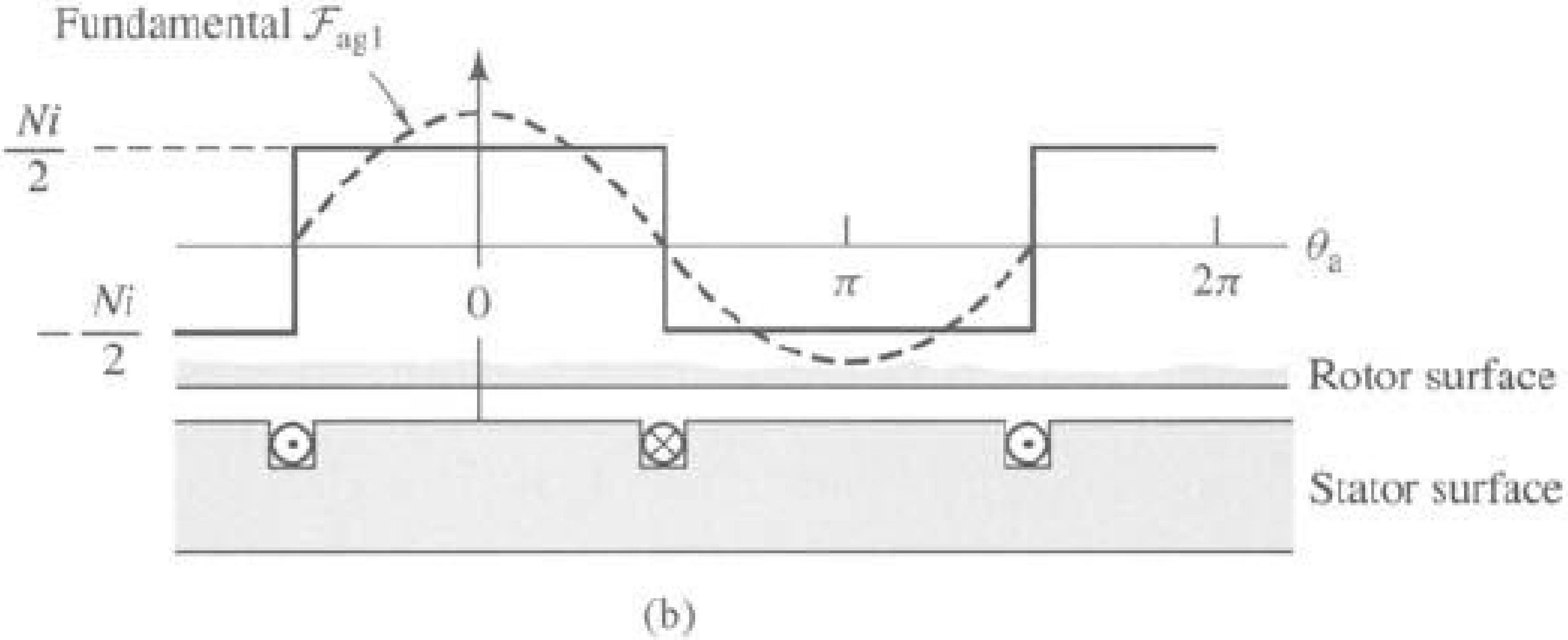


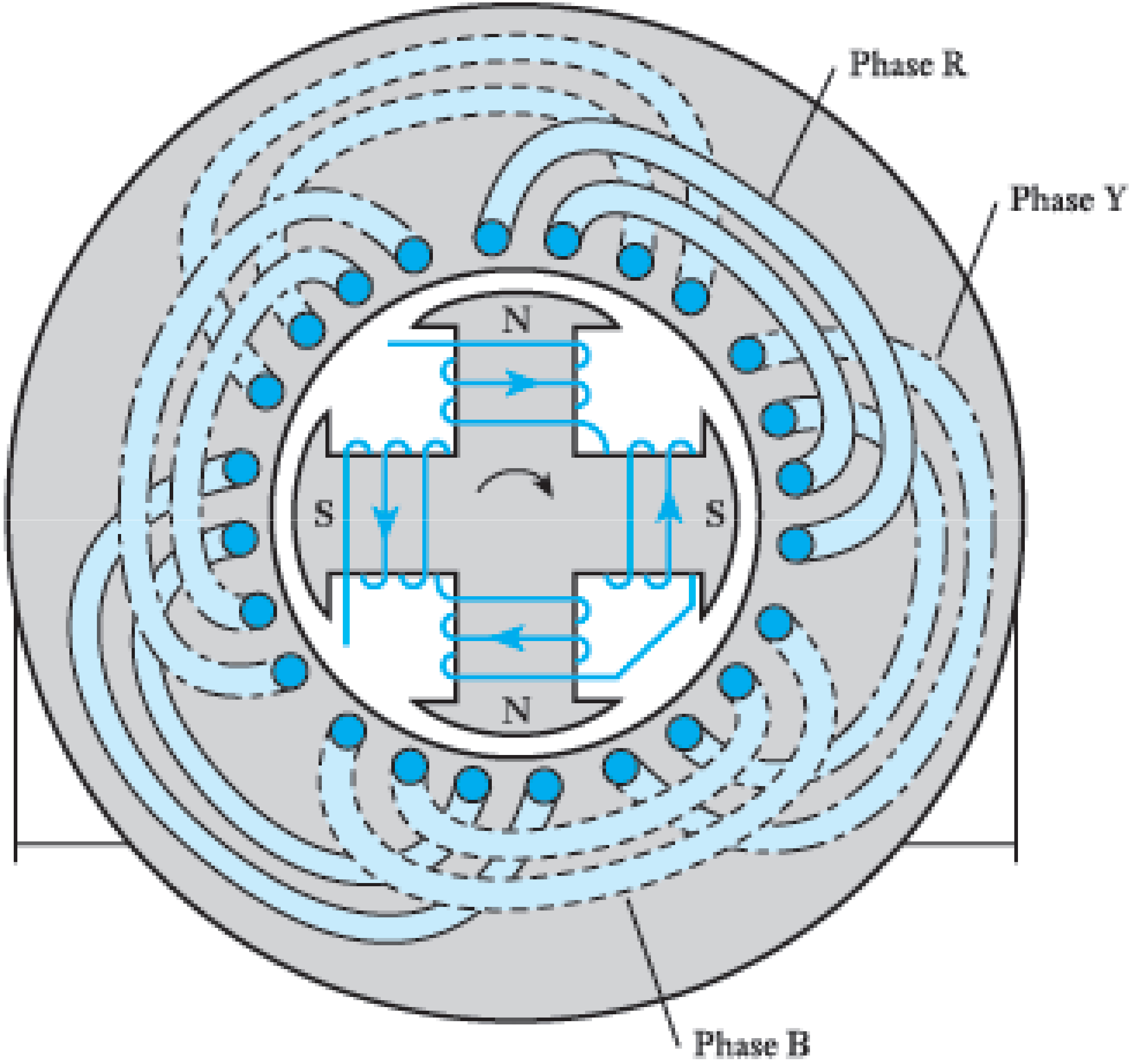
Figure 4.19 (a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap. (b) The air-gap mmf produced by current in this winding.

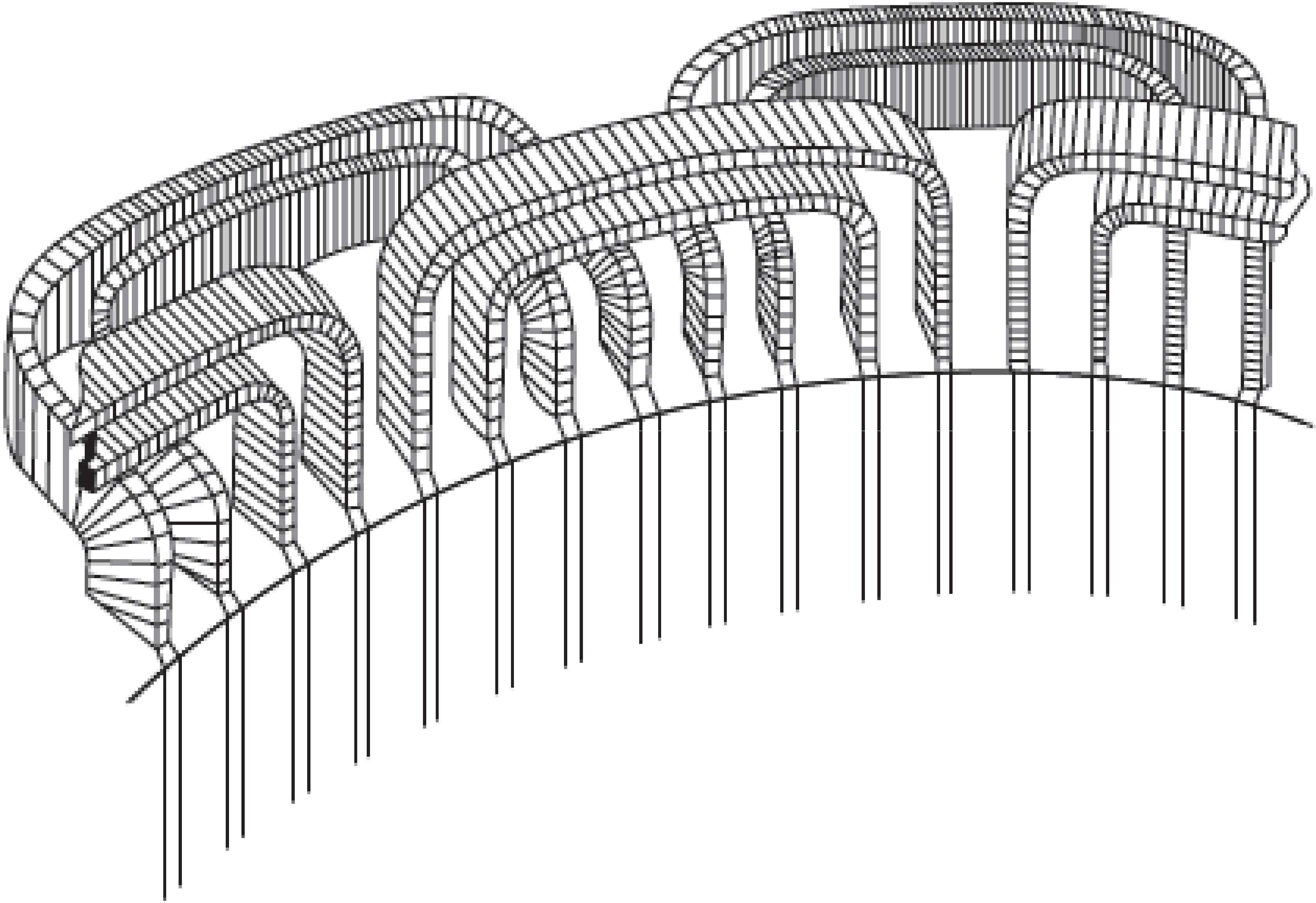
- *This MMF can be resolved by Fourier's series into a series of odd harmonics. The fundamental component is:*
- $$\mathcal{F}_{ag1} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta_a$$
- *This MMF wave is standing in space around the air-gap periphery, with its peak located along the magnetic axis of that coil.*
- $\mathcal{F}_{ag1} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta_a = \mathcal{F}_{peak}\ \cos\theta_a$

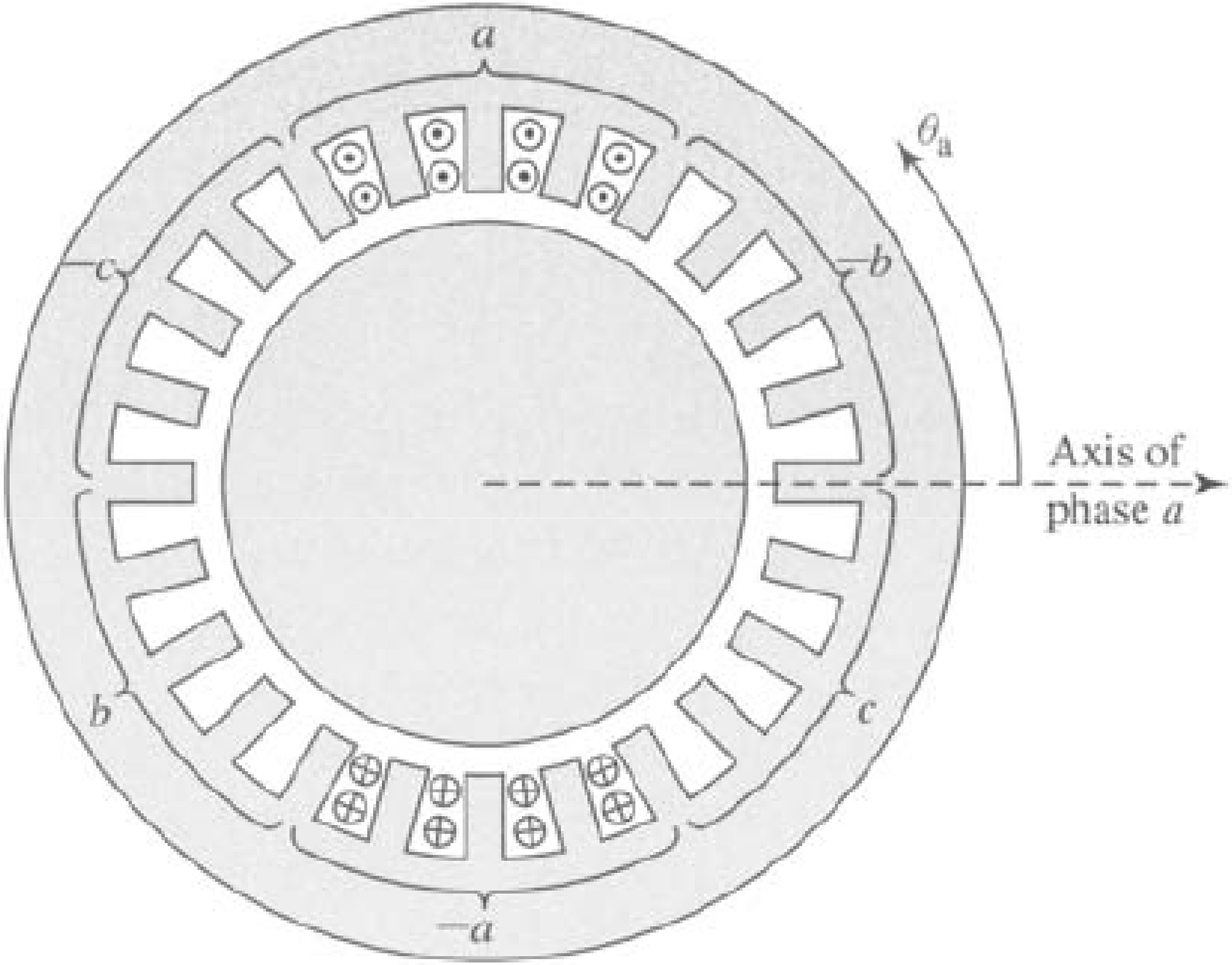
- *Where* ϑ_a *is the electrical angle measured from magnetic axis of phase a.*
- $\mathcal{F}_{peak}$ *is the peak MMF of phase a, which is always located along magnetic axis of phase a.*

Case-2: MMF of distributed winding

- *When the machine windings are distributed as usual along certain number of slots, the MMF distribution approximates more and more to sinusoidal wave form. See the figures below*


Phase R
Phase Y
N
S
S
N
Phase B





(a)

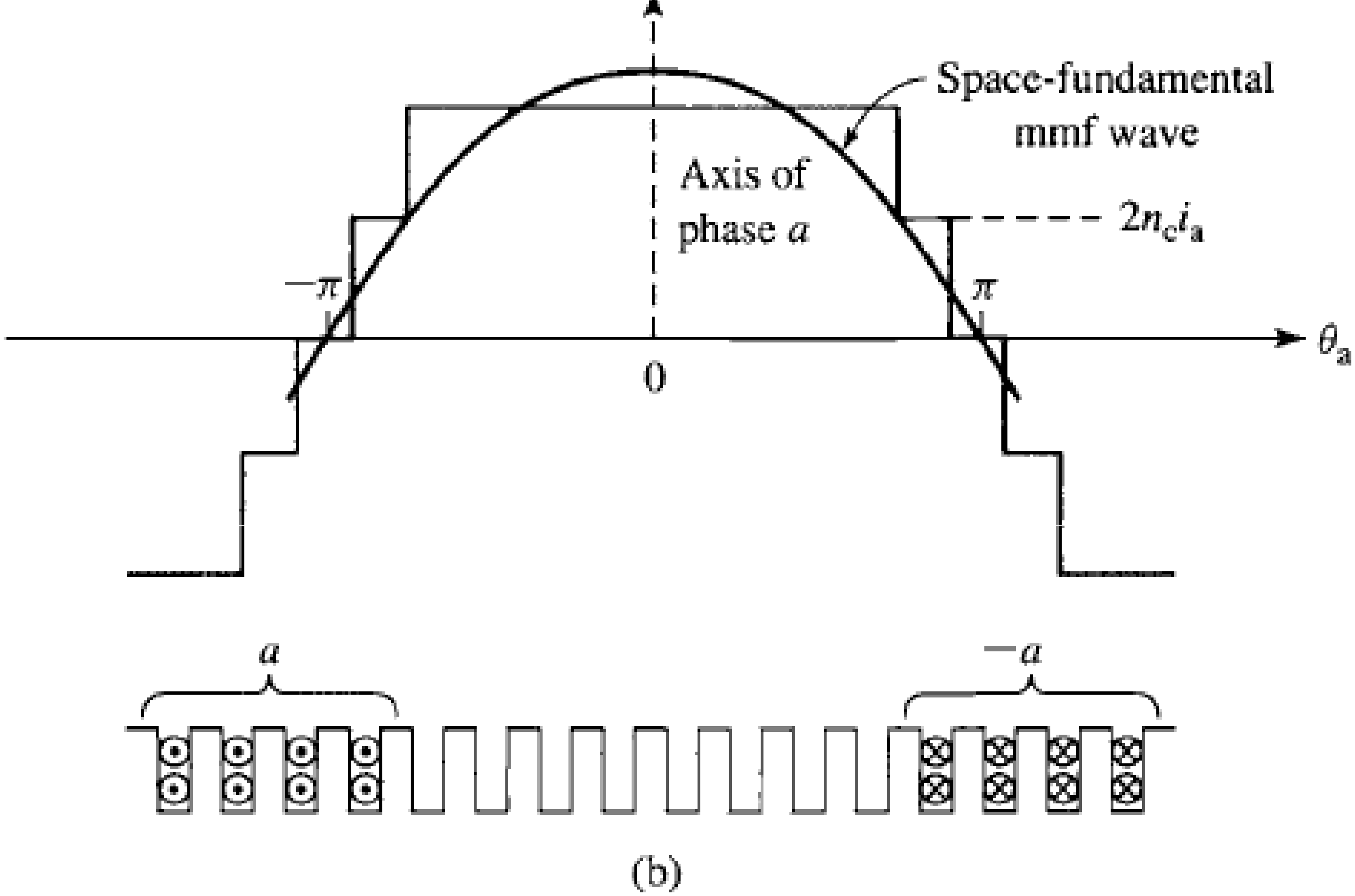


Figure 4.20 The mmf of one phase of a distributed two-pole, three-phase winding with full-pitch coils.

- *This MMF can be also resolved into a series of odd harmonics. The fundamental component has its peak value located along the magnetic axis of the winding. The space distribution of this fundamental MMf of phase a is given below.*
- $\mathcal{F}_{ag1} = \frac{4}{\pi}\left(\frac{K_w N_{ph}}{p}\right) i_a \cos\theta_a = \mathcal{F}_{apeak} \cos\left(\frac{p}{2}\ \theta_m\right)$

- *The factor 4/π arises from Fourier's series, and the factor* K_w *is known as the winding factor. It takes into account the distribution of armature windings, and also the chording of the winding.*
- Θ_m *is the mechanical angle measured from axis of phase a.*

$$\mathcal{F}_{\text{ag}1} = \frac{4}{\pi}\left(\frac{k_\text{w} N_\text{ph}}{\text{poles}}\right) i_\text{a} \cos\left(\frac{\text{poles}}{2}\theta_\text{a}\right) \tag{4.5}$$

in which the factor $4/\pi$ arises from the Fourier-series analysis of the rectangular mmf wave of a concentrated full-pitch coil, as in Eq. 4.3, and the *winding factor* k_w takes into account the distribution of the winding. This factor is required because the mmf's produced by the individual coils of any one phase group have different magnetic axes.

The factor $k_\text{w} N_\text{ph}$ is the effective series turns per phase for the fundamental mmf. The peak amplitude of this mmf wave is

$$(F_{\text{ag}1})_{\text{peak}} = \frac{4}{\pi}\left(\frac{k_\text{w} N_\text{ph}}{\text{poles}}\right) i_\text{a} \tag{4.6}$$

- *Also for the other phases we can write the MMF equations in similar forms as:*
- $\mathcal{F}_{bg1} = \frac{4}{\pi}\left(\frac{K_w N_{ph}}{p}\right) i_b \cos(\theta_a - 120)$
- $\mathcal{F}_{cg1} = \frac{4}{\pi}\left(\frac{K_w N_{ph}}{p}\right) i_c \cos(\theta_a + 120)$

- *Note that the peak MMF of each phase is proportional to the instantaneous value of phase current, and is always located along the magnetic axis of that phase.*
- *The result is three standing pulsating MMF waves, with each wave pulsating about its magnetic axis.*

Case-3: MMF of dc machines

- *In dc machines, the armature windings are placed on the rotor. They are distributed uniformly among slots on the external surface of the rotor as shown.*

DC. Machine Construction

2- Rotor: العضو الدوار

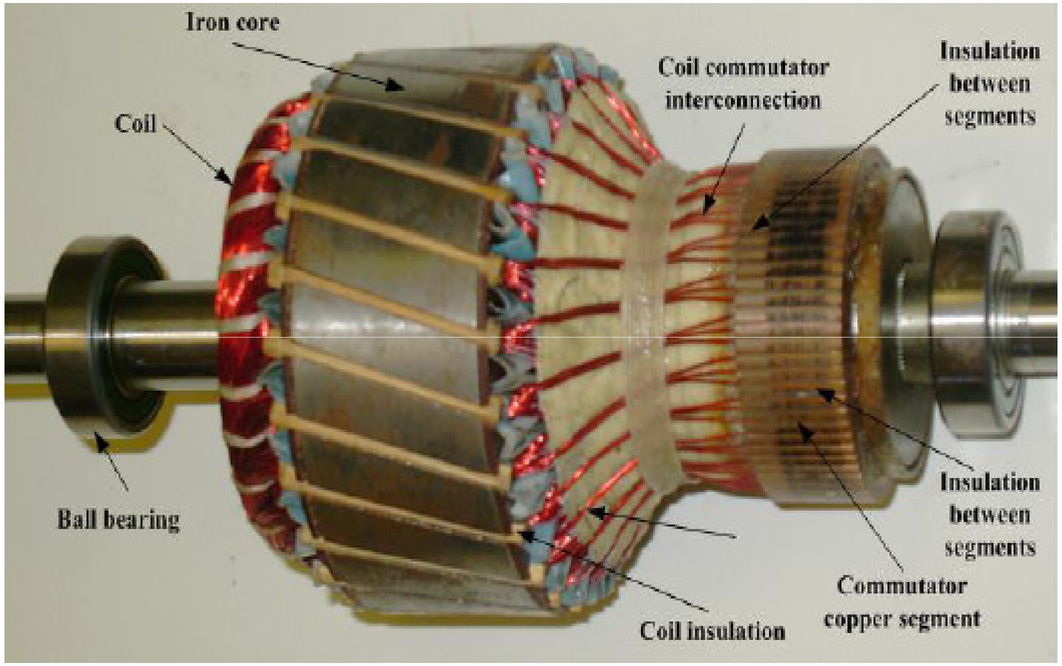
ويشمل:

shaftعمود الدوران

Armature Windingsملفات المنتج

قطاعات عضو التوحيد

commutator segments


Iron core
Coil
Coil commutator interconnection
Insulation between segments
Ball bearing
Insulation between segments
Commutator copper segment
Coil insulation

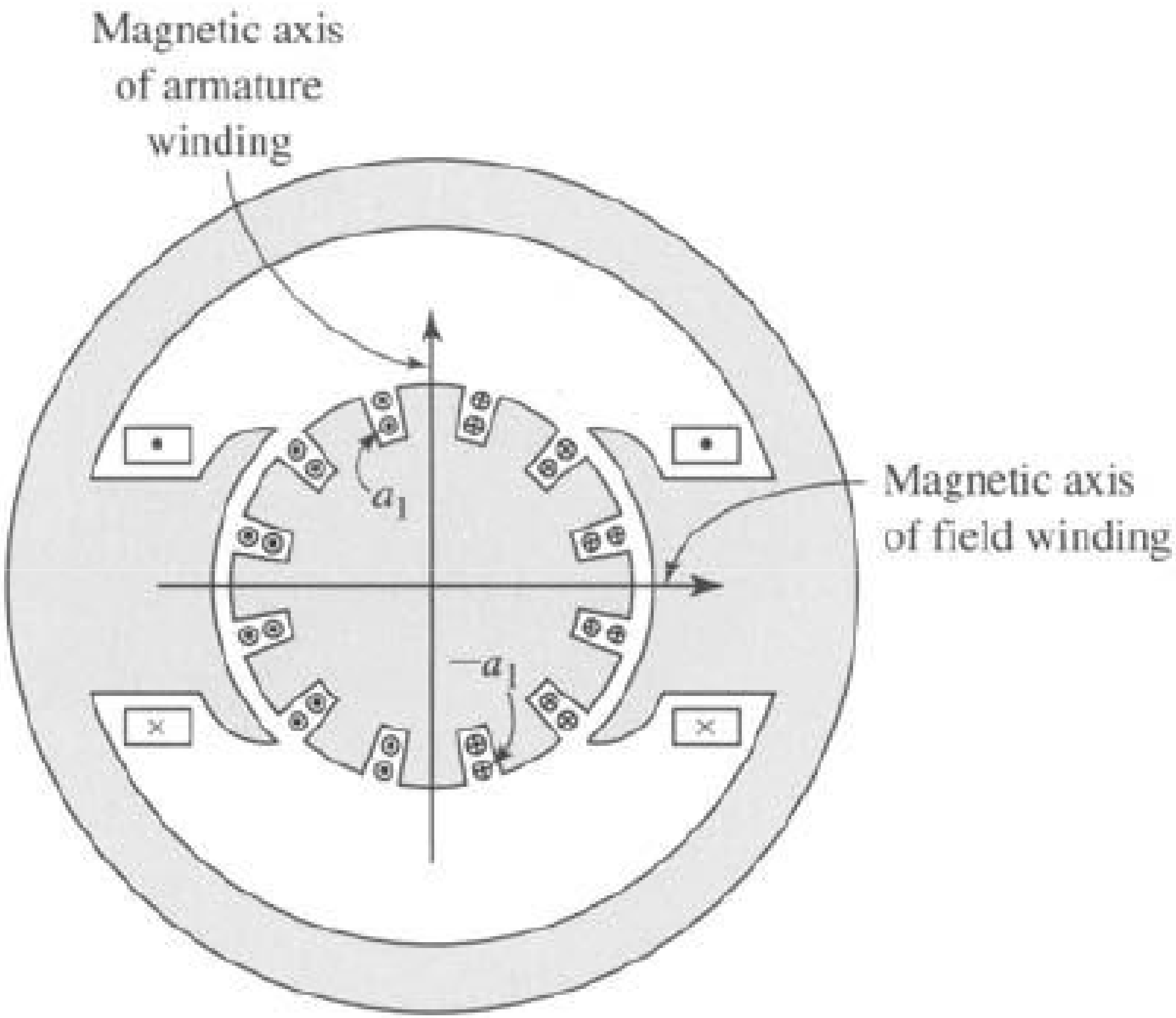
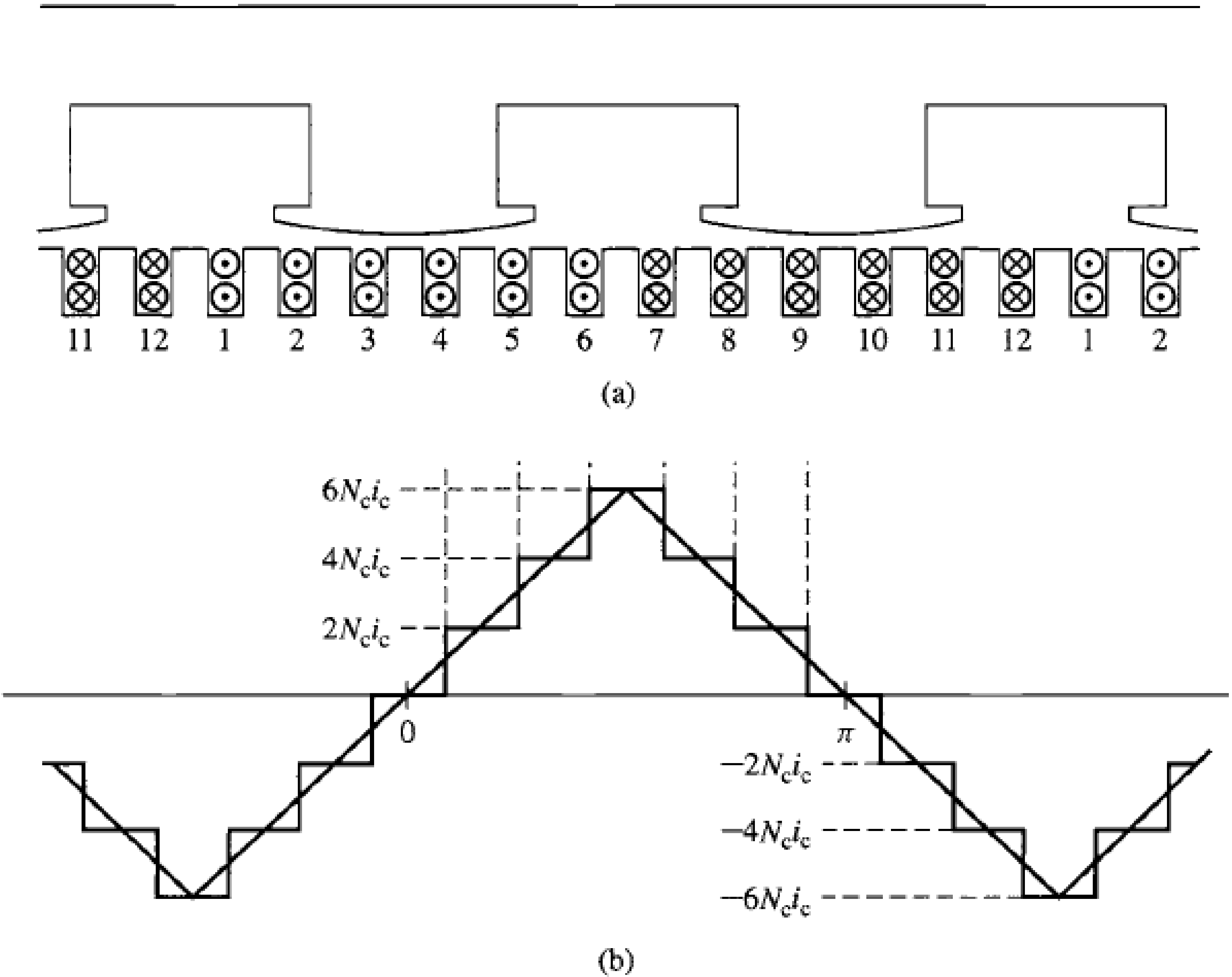
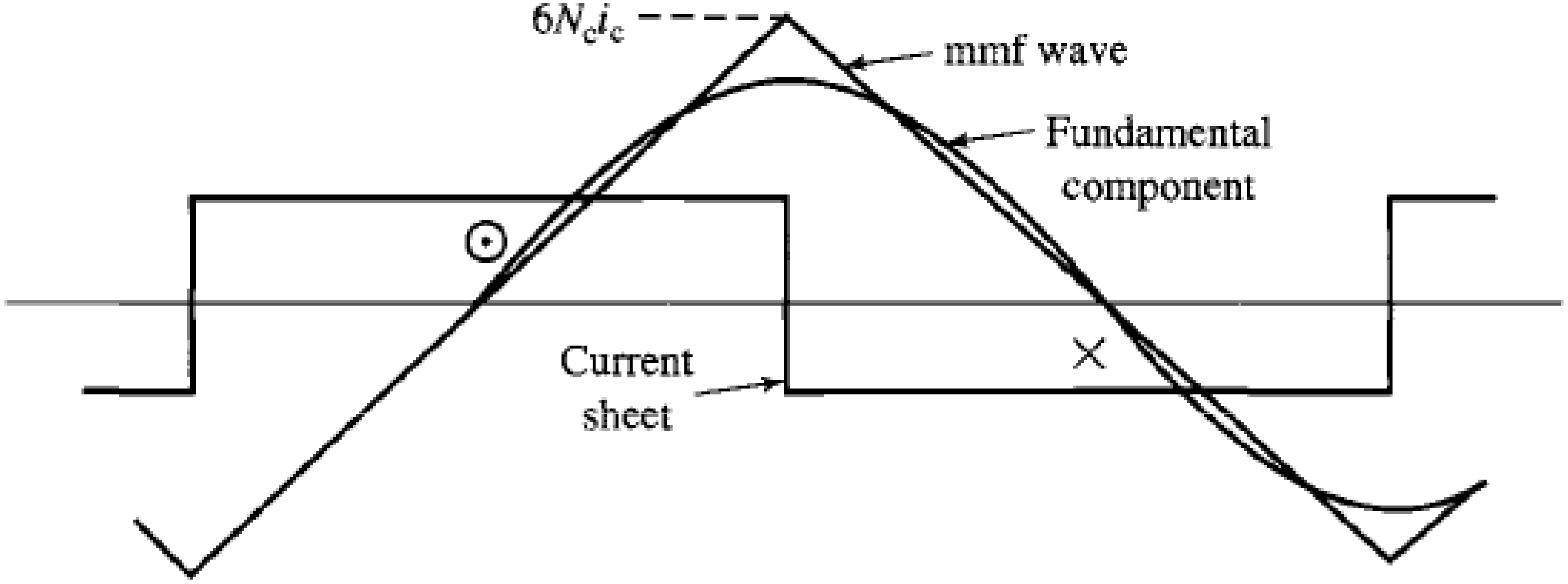

Magnetic axis of armature winding
a_1
Magnetic axis of field winding
$-a_1$

Figure 4.22 Cross section of a two-pole dc machine.


(a)
$6N_c i_c$
$4N_c i_c$
$2N_c i_c$
π
$-2N_c i_c$
$-4N_c i_c$
$-6N_c i_c$
(b)



(c)

Figure 4.23 (a) Developed sketch of the dc machine of Fig. 4.22; (b) mmf wave; (c) equivalent sawtooth mmf wave, its fundamental component, and equivalent rectangular current sheet.

The peak value of the sawtooth armature mmf wave can be written in terms of the total number of conductors in the armature slots as

$$(F_{\mathrm{ag}})_{\mathrm{peak}} = \left(\frac{C_{\mathrm{a}}}{2m \cdot \mathrm{poles}}\right) i_{\mathrm{a}} \qquad \mathrm{A} \cdot \mathrm{turns/pole} \tag{4.9}$$

where

C_{a} = total number of conductors in armature winding
m = number of parallel paths through armature winding
i_{a} = armature current, A

$$(F_{\mathrm{ag}})_{\mathrm{peak}} = \left(\frac{N_{\mathrm{a}}}{\mathrm{poles}}\right) i_{\mathrm{a}}$$

where $N_{\mathrm{a}} = C_{\mathrm{a}}/(2m)$ is the number of series armature turns. From the Fourier series for the sawtooth mmf wave of Fig. 4.24b, the peak value of the space fundamental is given by

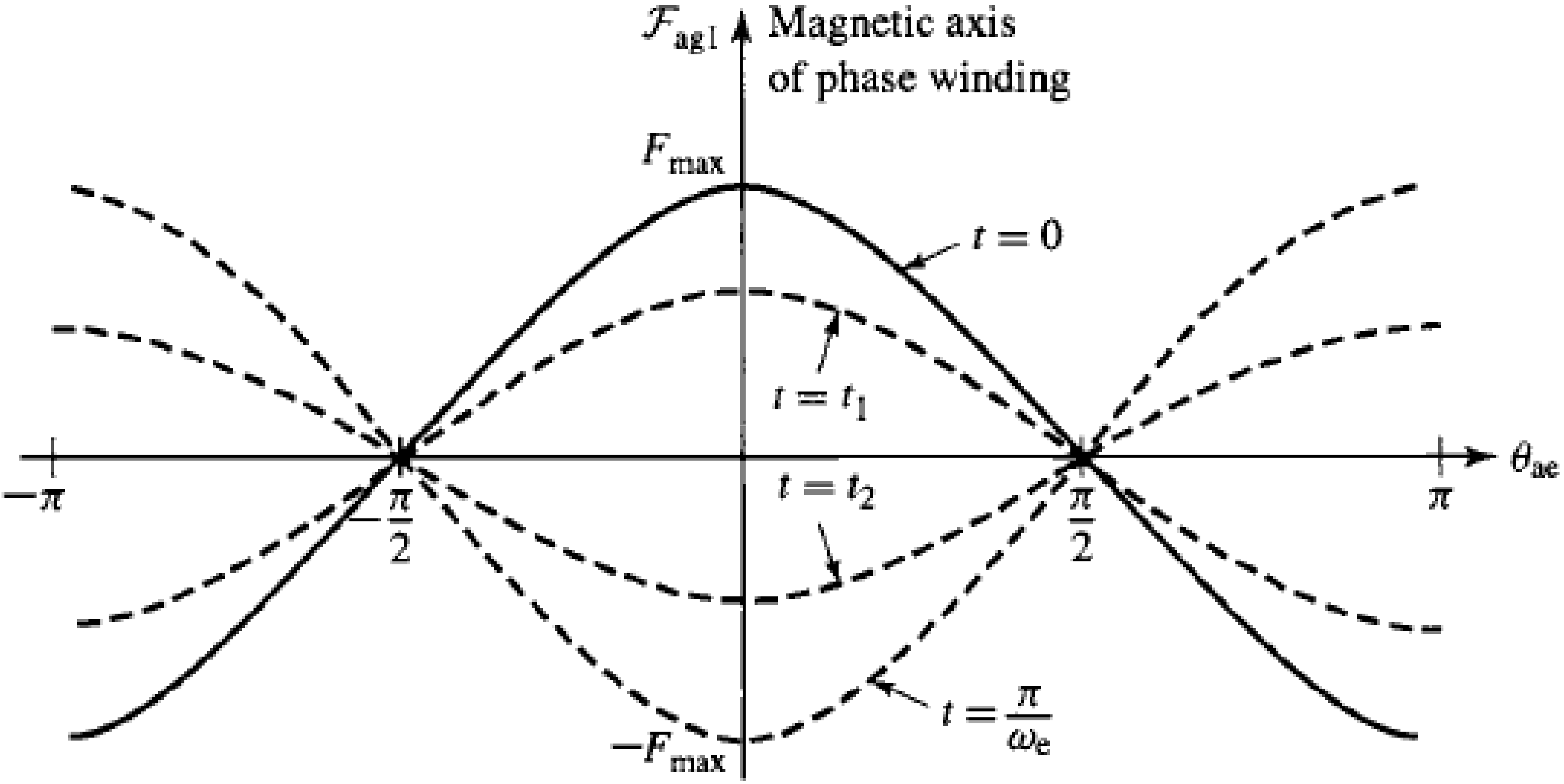
$$(F_{\mathrm{ag1}})_{\mathrm{peak}} = \frac{8}{\pi^2}\left(\frac{N_{\mathrm{a}}}{\mathrm{poles}}\right) i_{\mathrm{a}} \tag{4.11}$$

- The peak value of the sawtooth wave form can be written as:
- $\left(\mathcal{F}_{ag1}\right)_{peak} = \left(\frac{c_a}{2m\,p}\right) I_a$ A.turn/pole
- Where:
- C_a = total number of conductors in the armature.
- m = number of parallel paths in which the armature windings are connected.
- I_a = armature current A.

- *Then from Fourier's series, the sawtooth wave form can be resolved into series of odd harmonics. The fundamental component is:*
- $\mathcal{F}_{ag1} = \frac{8}{\pi^2}\left(\frac{N_a}{p}\right) I_a \cos\theta = \left(\mathcal{F}_{ag1}\right)_{peak} \cos\theta$

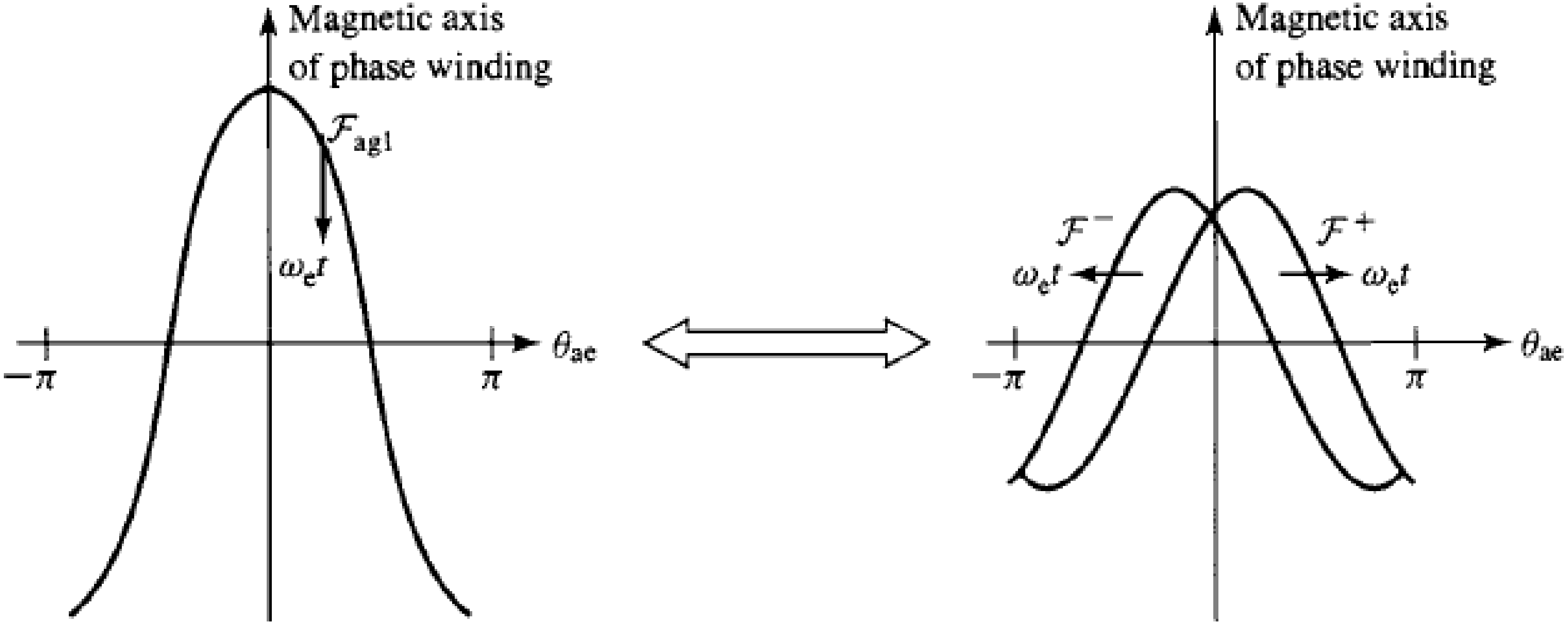
Rotating magnetic fields

- *1- Single phase machines: The previous standing pulsating mmf wave obtained for distributed windings can be resolved into two rotating mmf waves. One mmf wave rotates forward at synchronous speed, while the other rotates backward at synchronous speed too.*

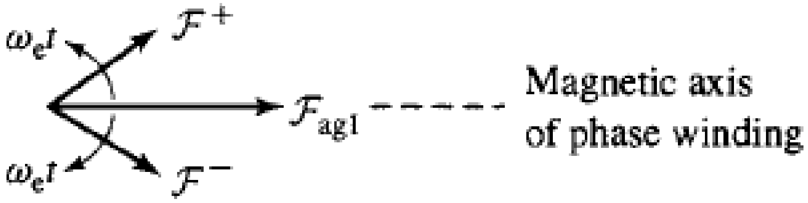


$\mathcal{F}_{\mathrm{ag1}}$
Magnetic axis of phase winding
$F_{\max}$
$t = 0$
$t = t_1$
$t = t_2$
$t = \frac{\pi}{\omega_e}$
$-F_{\max}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
θ_{ae}

(a)


Magnetic axis of phase winding
$\mathcal{F}_{ag1}$
$\omega_e t$
θ_{ae}
$-\pi$
π
Magnetic axis of phase winding
$\mathcal{F}^-$
$\mathcal{F}^+$
$\omega_e t$
$\omega_e t$
θ_{ae}
$-\pi$
π

(b)



(c)

Figure 4.28 Single-phase-winding space-fundamental air-gap mmf: (a) mmf distribution of a single-phase winding at various times; (b) total mmf $\mathcal{F}_{ag1}$ decomposed into two traveling waves $\mathcal{F}^-$ and $\mathcal{F}^+$; (c) phasor decomposition of $\mathcal{F}_{ag1}$.

the mmf distribution is given by

$$\begin{aligned}\mathcal{F}_{\text{ag1}} &= F_{\max} \cos\left(\frac{\text{poles}}{2}\theta_{\text{a}}\right) \cos \omega_{\text{e}} t \\ &= F_{\max} \cos(\theta_{\text{ae}}) \cos \omega_{\text{e}} t \qquad (4.18)\end{aligned}$$

Equation 4.18 has been written in a form to emphasize the fact that the result is an mmf distribution of maximum amplitude.

$$F_{\max} = \frac{4}{\pi}\left(\frac{k_{\text{w}} N_{\text{ph}}}{\text{poles}}\right) I_{\text{a}} \qquad (4.19)$$

$$\mathcal{F}_{\text{ag1}} = F_{\max}\left[\frac{1}{2}\cos(\theta_{\text{ae}} - \omega_{\text{e}} t) + \frac{1}{2}\cos(\theta_{\text{ae}} + \omega_{\text{e}} t)\right]$$

$$\mathcal{F}_{\text{ag1}}^{+} = \frac{1}{2} F_{\max} \cos(\theta_{\text{ae}} - \omega_{\text{e}} t)$$

$$\mathcal{F}_{\text{ag1}}^{-} = \frac{1}{2} F_{\max} \cos(\theta_{\text{ae}} + \omega_{\text{e}} t)$$

In single-phase ac machinery, the positive-traveling flux wave produces useful torque while the negative-traveling flux wave produces both negative and pulsating torque as well as losses. These machines are designed so as to minimize the effects of the negative-traveling flux wave, although, unlike in the case of polyphase machinery, these effects cannot be totally eliminated.

- *2- MMF of three phase machines:*
- *Each phase of the three phase machine produces its own MMF distribution. This MMF has a fundamental component as found earlier with its peak always located along the magnetic axis of that phase. This mmf can also be resolved into two rotating components, one rotates forward in the direction of phase sequence, while the other rotates backward in opposite direction to phase sequence.*

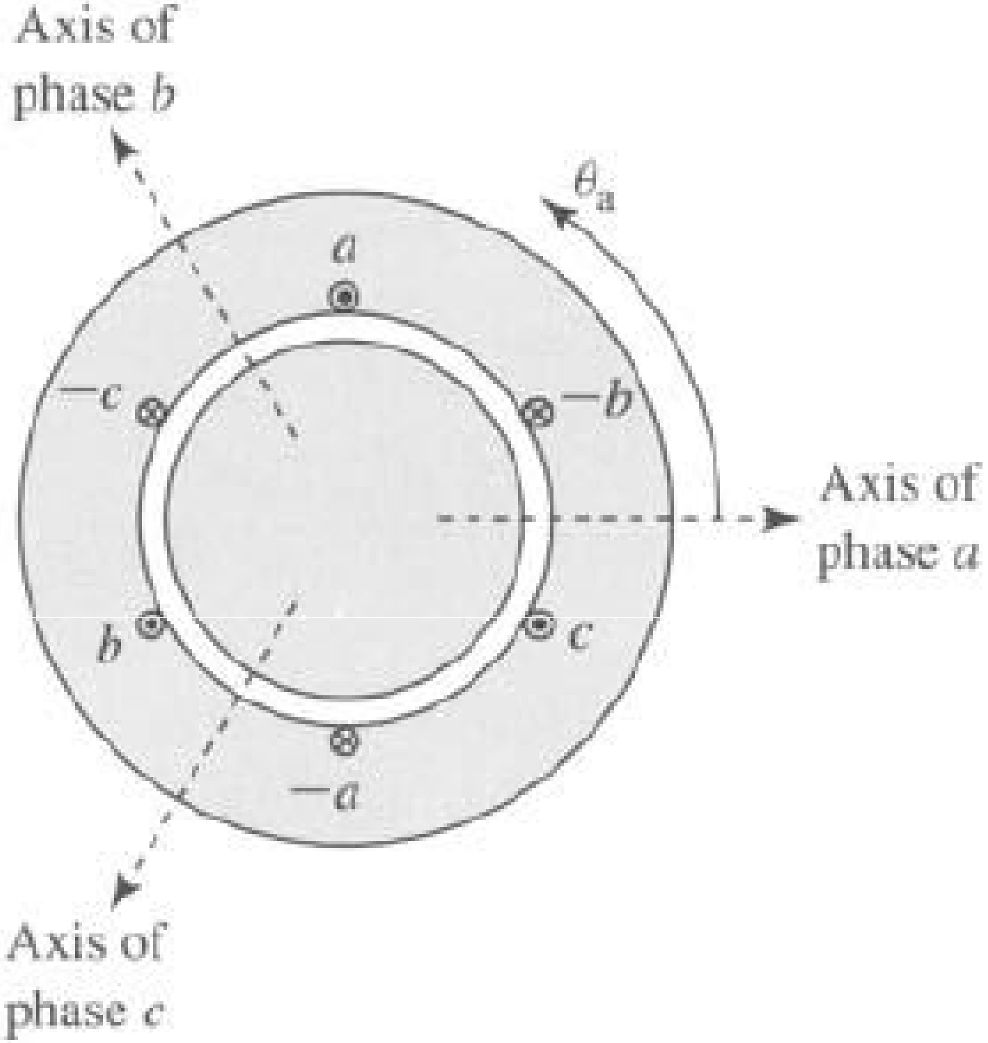


Figure 4.29 Simplified two-pole three-phase stator winding.

the instantaneous currents are

$$i_a = I_m \cos \omega_e t$$

$$i_b = I_m \cos (\omega_e t - 120^\circ)$$

$$i_c = I_m \cos (\omega_e t + 120^\circ)$$

The mmf of phase *a* has been shown to be

$$\mathcal{F}_{a1} = \mathcal{F}_{a1}^{+} + \mathcal{F}_{a1}^{-}$$

where

$$\mathcal{F}_{a1}^{+} = \frac{1}{2} F_{max} \cos (\theta_{ae} - \omega_e t)$$

$$\mathcal{F}_{a1}^{-} = \frac{1}{2} F_{max} \cos (\theta_{ae} + \omega_e t)$$

and

$$F_{max} = \frac{4}{\pi} \left(\frac{k_w N_{ph}}{\text{poles}} \right) I_m$$

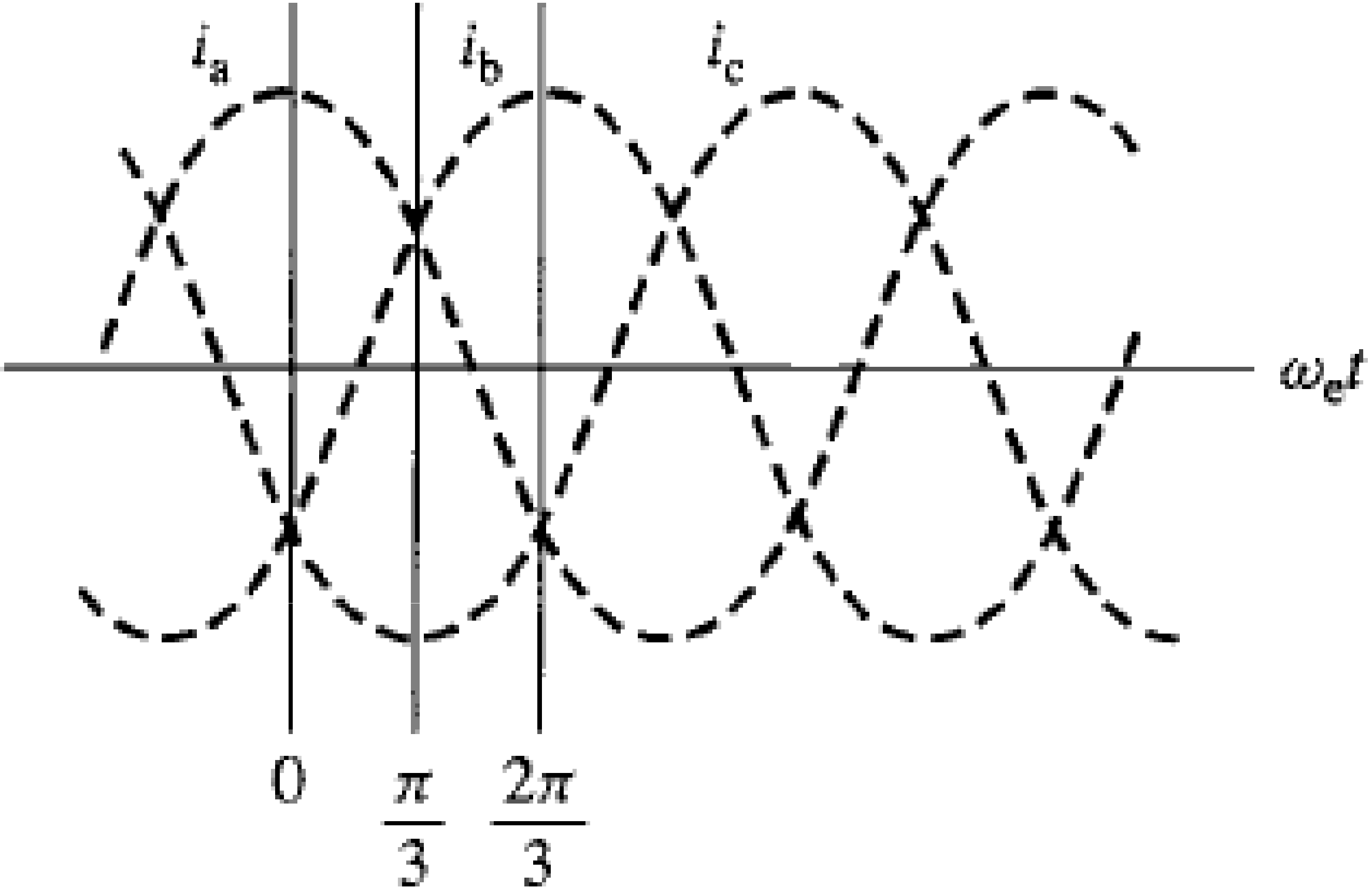


Figure 4.30 Instantaneous phase currents under balanced three-phase conditions.

Similarly, for phases b and c, whose axes are at $\theta_a = 120°$ and $\theta_a = -120°$, respectively,

$$\mathcal{F}_{b1} = \mathcal{F}_{b1}^{+} + \mathcal{F}_{b1}^{-} \tag{4.30}$$

$$\mathcal{F}_{b1}^{+} = \frac{1}{2} F_{\max} \cos\left(\theta_{ae} - \omega_e t\right) \tag{4.31}$$

$$\mathcal{F}_{b1}^{-} = \frac{1}{2} F_{\max} \cos\left(\theta_{ae} + \omega_e t + 120°\right) \tag{4.32}$$

and

$$\mathcal{F}_{c1} = \mathcal{F}_{c1}^{+} + \mathcal{F}_{c1}^{-} \tag{4.33}$$

$$\mathcal{F}_{c1}^{+} = \frac{1}{2} F_{\max} \cos\left(\theta_{ae} - \omega_e t\right) \tag{4.34}$$

$$\mathcal{F}_{c1}^{-} = \frac{1}{2} F_{\max} \cos\left(\theta_{ae} + \omega_e t - 120°\right) \tag{4.35}$$

The total mmf is the sum of the contributions from each of the three phases

$$\mathcal{F}(\theta_{ae}, t) = \mathcal{F}_{a1} + \mathcal{F}_{b1} + \mathcal{F}_{c1} \tag{4.36}$$

This summation can be performed quite easily in terms of the positive- and negative-traveling waves. The negative-traveling waves sum to zero

$$\begin{aligned}\mathcal{F}^-(\theta_{ae}, t) &= \mathcal{F}^-_{a1} + \mathcal{F}^-_{b1} + \mathcal{F}^-_{c1} \\ &= \frac{1}{2} F_{max} [\cos(\theta_{ae} + \omega_e t) + \cos(\theta_{ae} + \omega_e t - 120°) \\ &\quad + \cos(\theta_{ae} + \omega_e t + 120°)] \\ &= 0 \end{aligned} \tag{4.37}$$

while the positive-traveling waves reinforce

$$\begin{aligned}\mathcal{F}^+(\theta_{ae}, t) &= \mathcal{F}^+_{a1} + \mathcal{F}^+_{b1} + \mathcal{F}^+_{c1} \\ &= \frac{3}{2} F_{max} \cos(\theta_{ae} - \omega_e t)\end{aligned}$$

Thus, the result of displacing the three windings by 120° in space phase and displacing the winding currents by 120° in time phase is a single positive-traveling mmf wave

$$\begin{aligned}\mathcal{F}(\theta_{ae}, t) &= \frac{3}{2} F_{max} \cos(\theta_{ae} - \omega_e t) \\ &= \frac{3}{2} F_{max} \cos\left(\left(\frac{\text{poles}}{2}\right)\theta_a - \omega_e t\right) \qquad (4.39)\end{aligned}$$

The air-gap mmf wave described by Eq. 4.39 is a space-fundamental sinusoidal function of the electrical space angle θ_{ae} (and hence of the space angle $\theta_a = (2/\text{poles})\theta_{ae}$). It has a constant amplitude of $(3/2)F_{max}$, i.e., 1.5 times the amplitude of the air-gap mmf wave produced by the individual phases alone. It has a positive peak at angle $\theta_a = (2/\text{poles})\omega_e t$. Thus, under balanced three-phase conditions, the three-phase winding produces an air-gap mmf wave which rotates at *synchronous angular velocity* ω_s

$$\omega_s = \left(\frac{2}{\text{poles}}\right)\omega_e \tag{4.40}$$

where

ω_e = angular frequency of the applied electrical excitation [rad/sec]
ω_s = synchronous spatial angular velocity of the air-gap mmf wave [rad/sec]

The corresponding *synchronous speed* n_s in r/min can be expressed in terms of the applied electrical frequency $f_e = \omega_e/(2\pi)$ in Hz as

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e \quad \text{r/min} \tag{4.41}$$

How to establish a rotating magnetic field

- *In order to establish a rotating magnetic field in electric machines, we should have:*
- *1- At least two windings that are space displaced from each other.*
- *2- The currents that flow in these windings should be displaced in time { in phase} by certain angle greater than zero.*
- *These conditions are easily fulfilled in two phase, and three phase machines.*

- *While in single phase machines, these conditions are obtained by:*
- *1- Split phase windings in space with capacitor placed in one of them to obtain the time phase.*
- *2- Split phase windings with shaded poles in one winding to obtain the phase difference.*

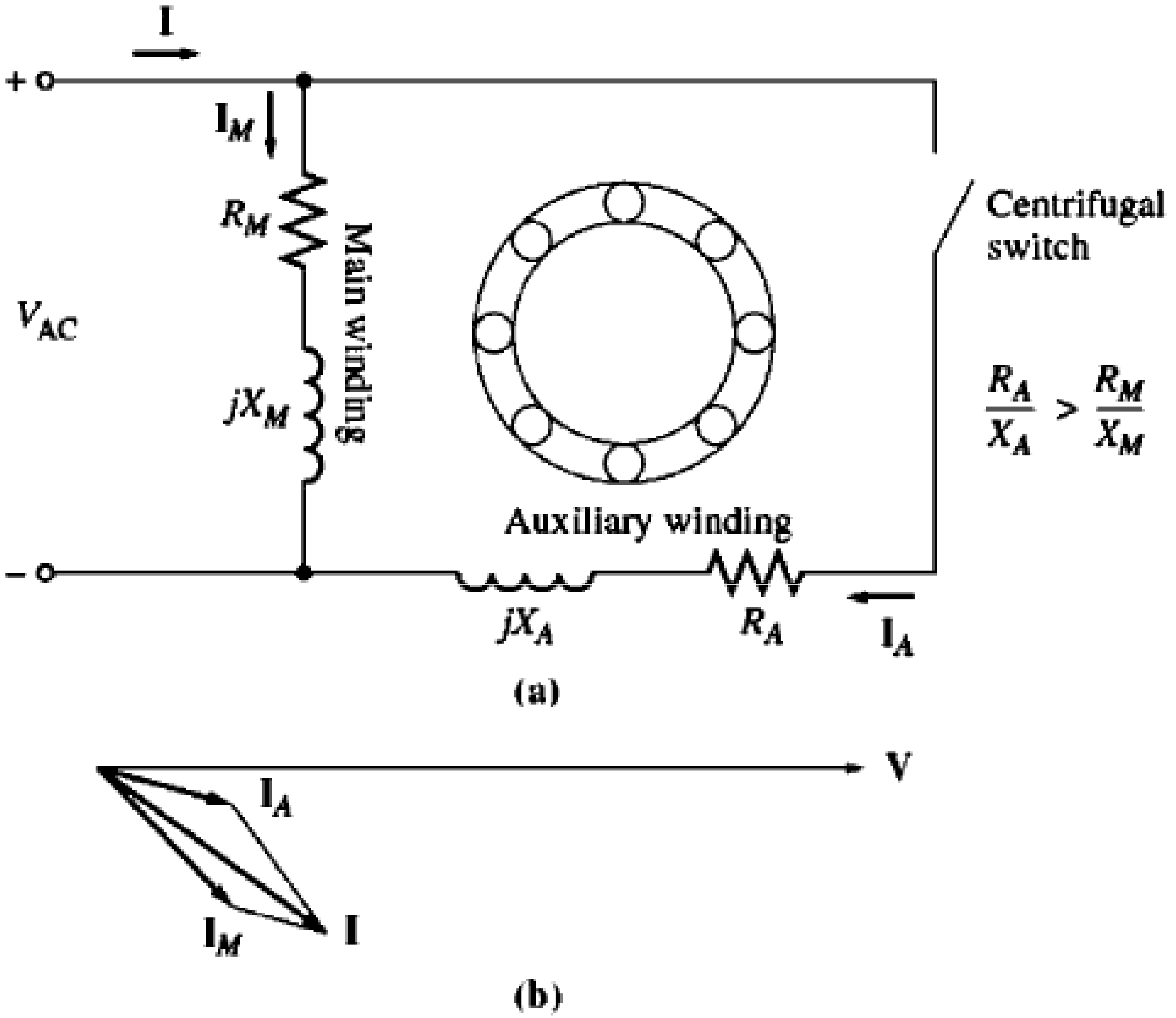
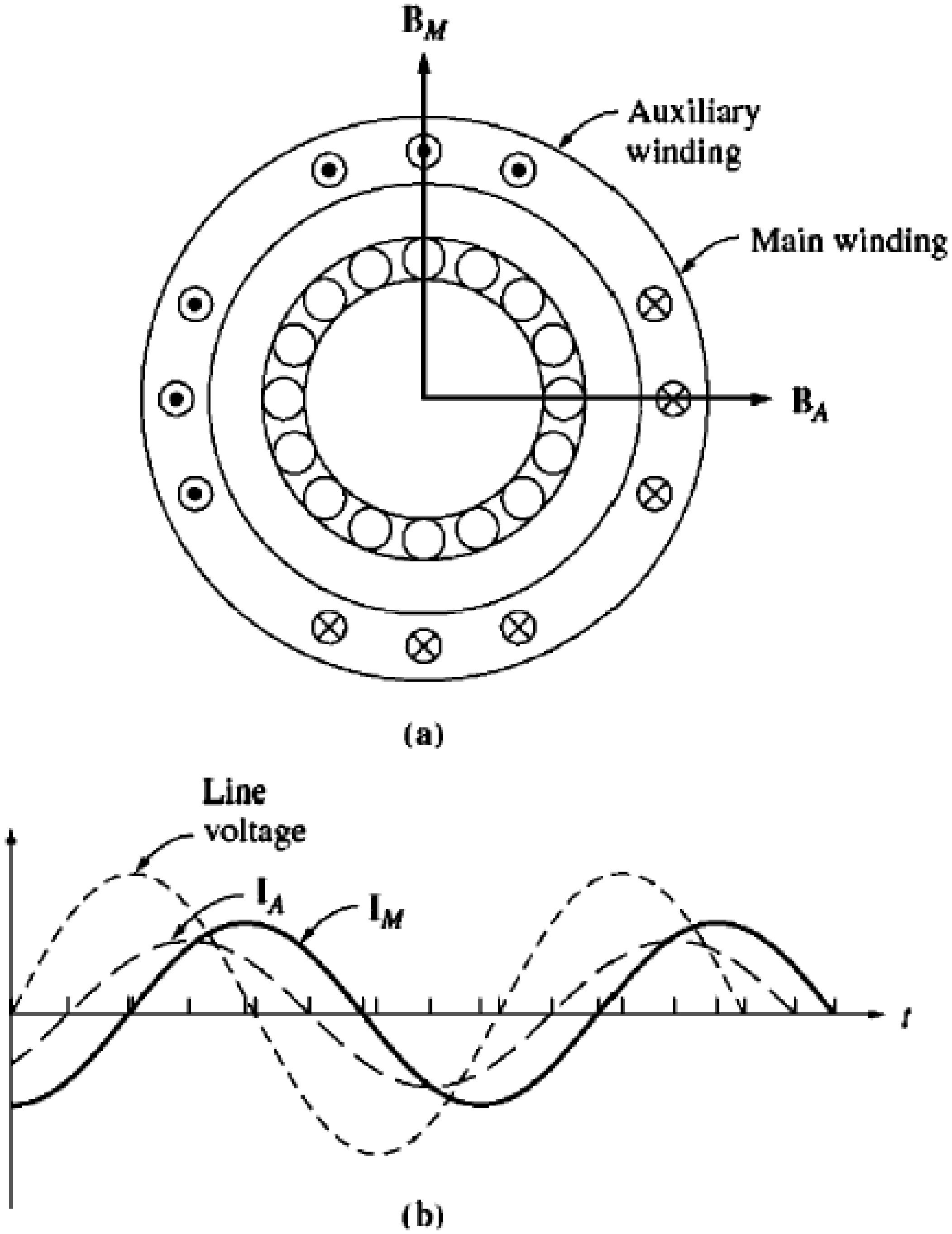


FIGURE 10–14
(a) A split-phase induction motor. (b) The currents in the motor at starting conditions.


B_M
Auxiliary winding
Main winding
B_A
(a)
Line voltage
I_A
I_M
t
(b)

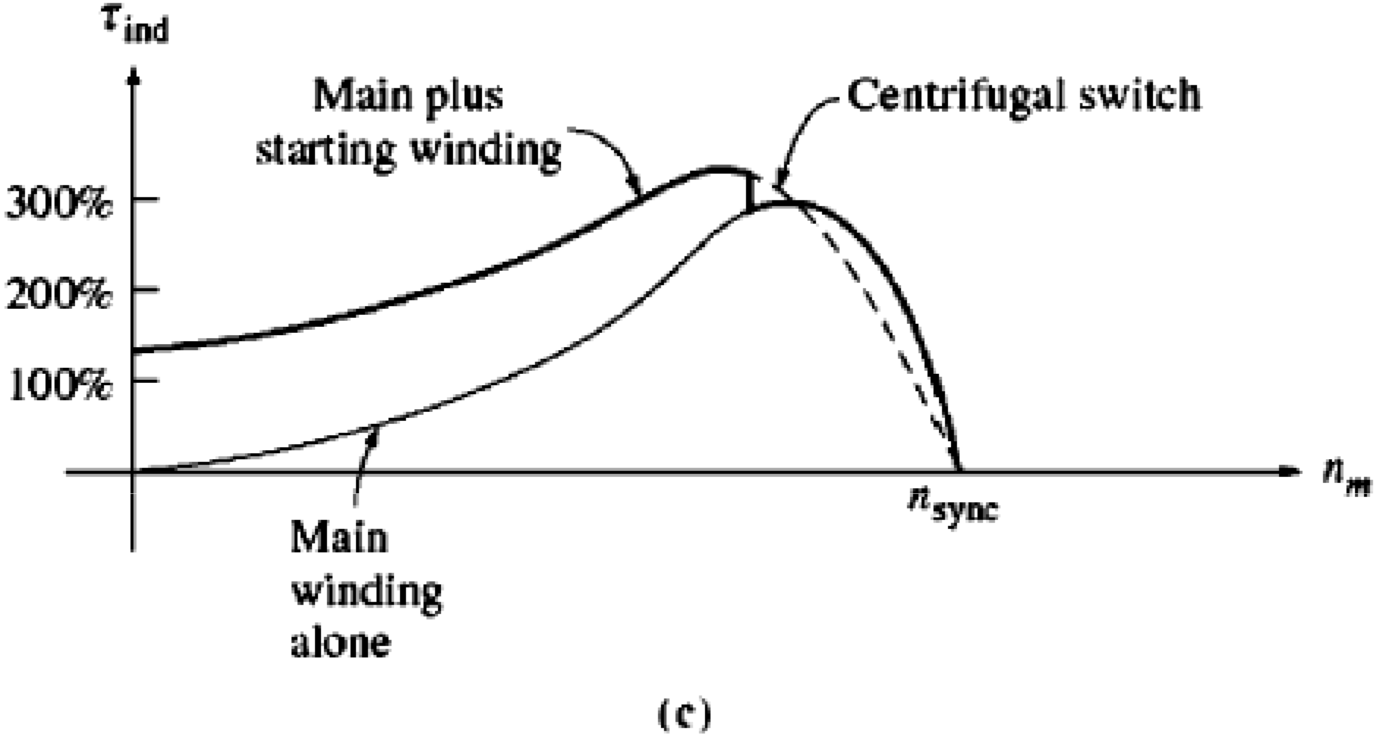
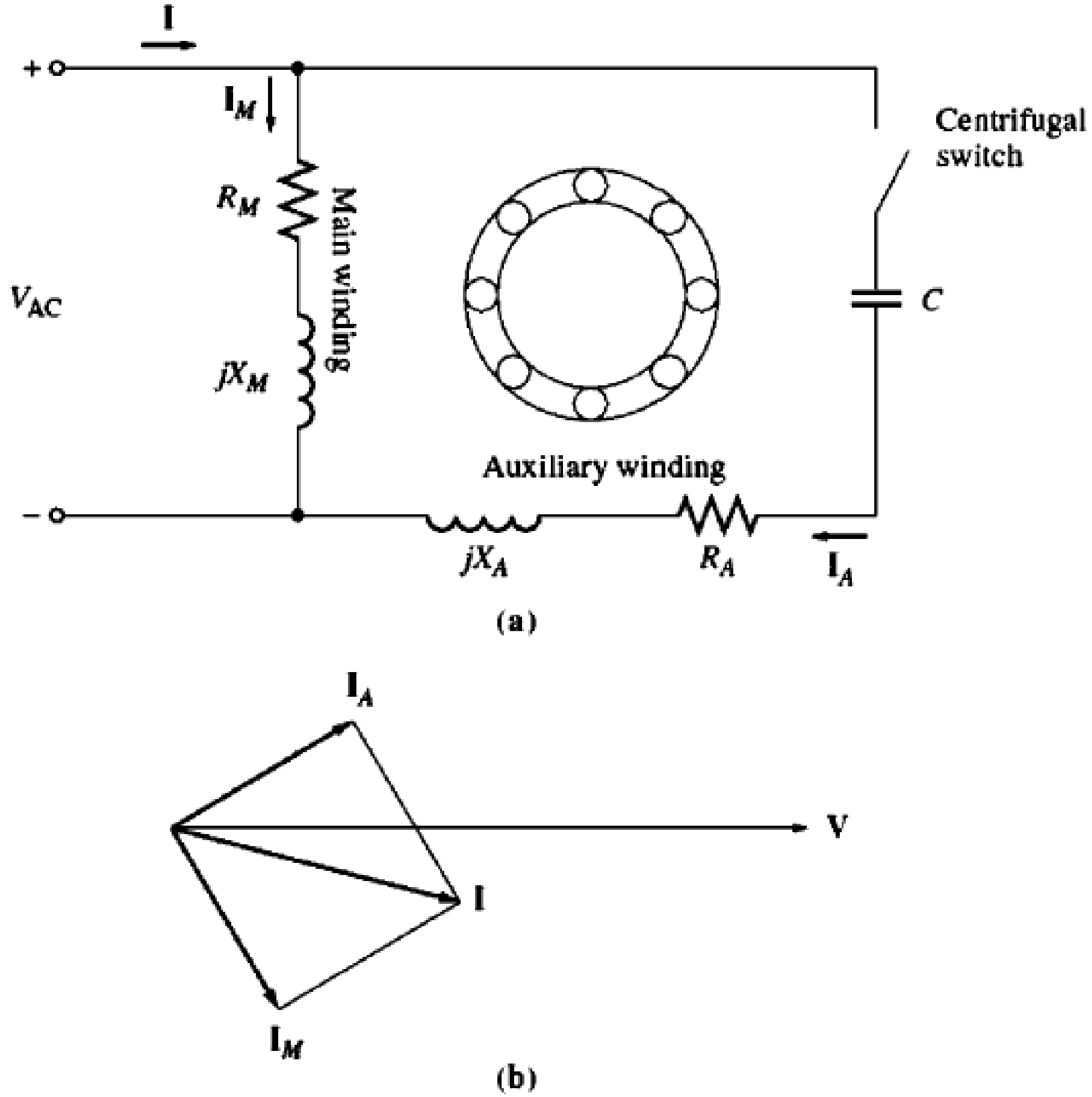


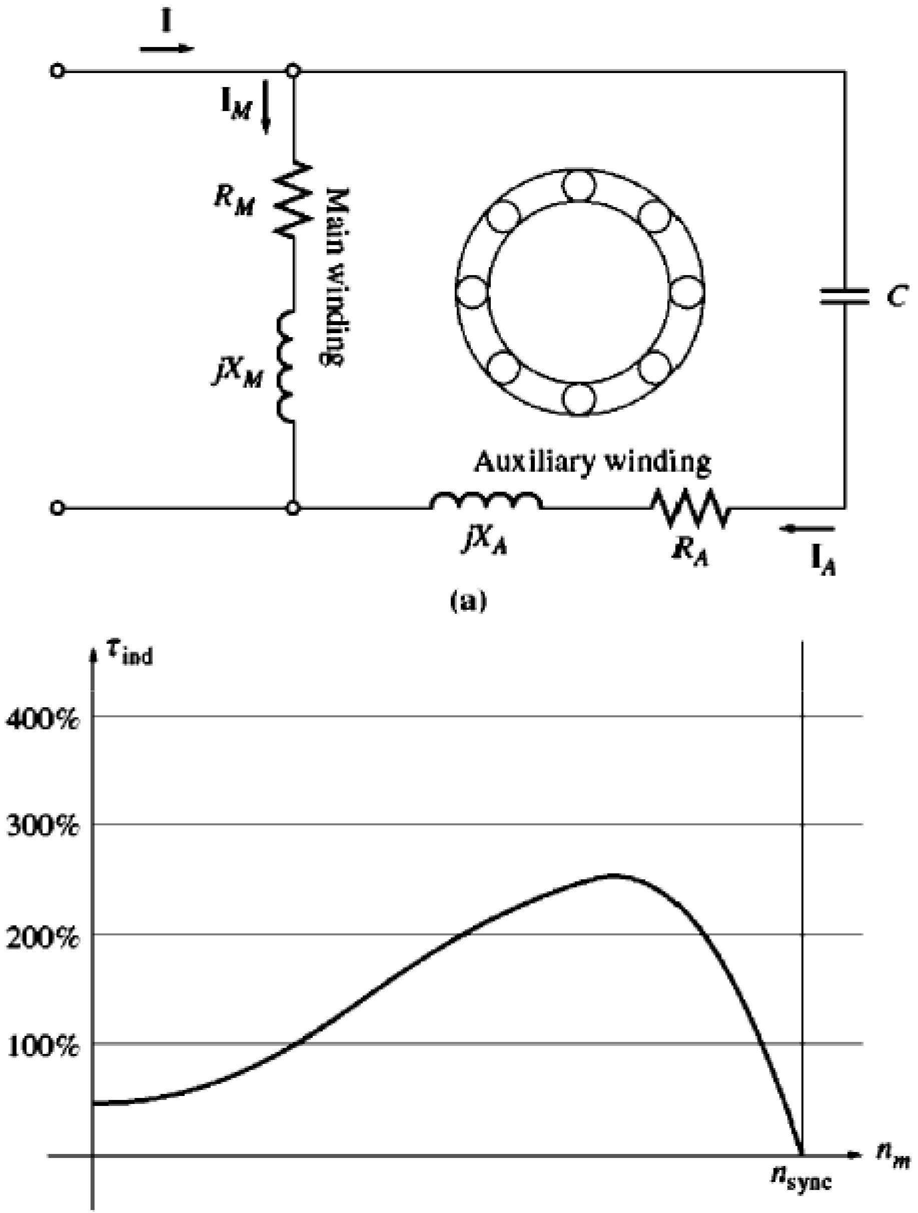
FIGURE 10–15
(a) Relationship of main and auxiliary magnetic fields. (b) $\mathbf{I}_A$ peaks before $\mathbf{I}_M$, producing a net counterclockwise rotation of the magnetic fields. (c) The resulting torque–speed characteristic.



(a)

(b)

FIGURE 10–17
(a) A capacitor-start induction motor. (b) Current angles at starting in this motor.



(b)

FIGURE 10–20
(a) A permanent split-capacitor induction motor. (b) Torque–speed characteristic of this motor.

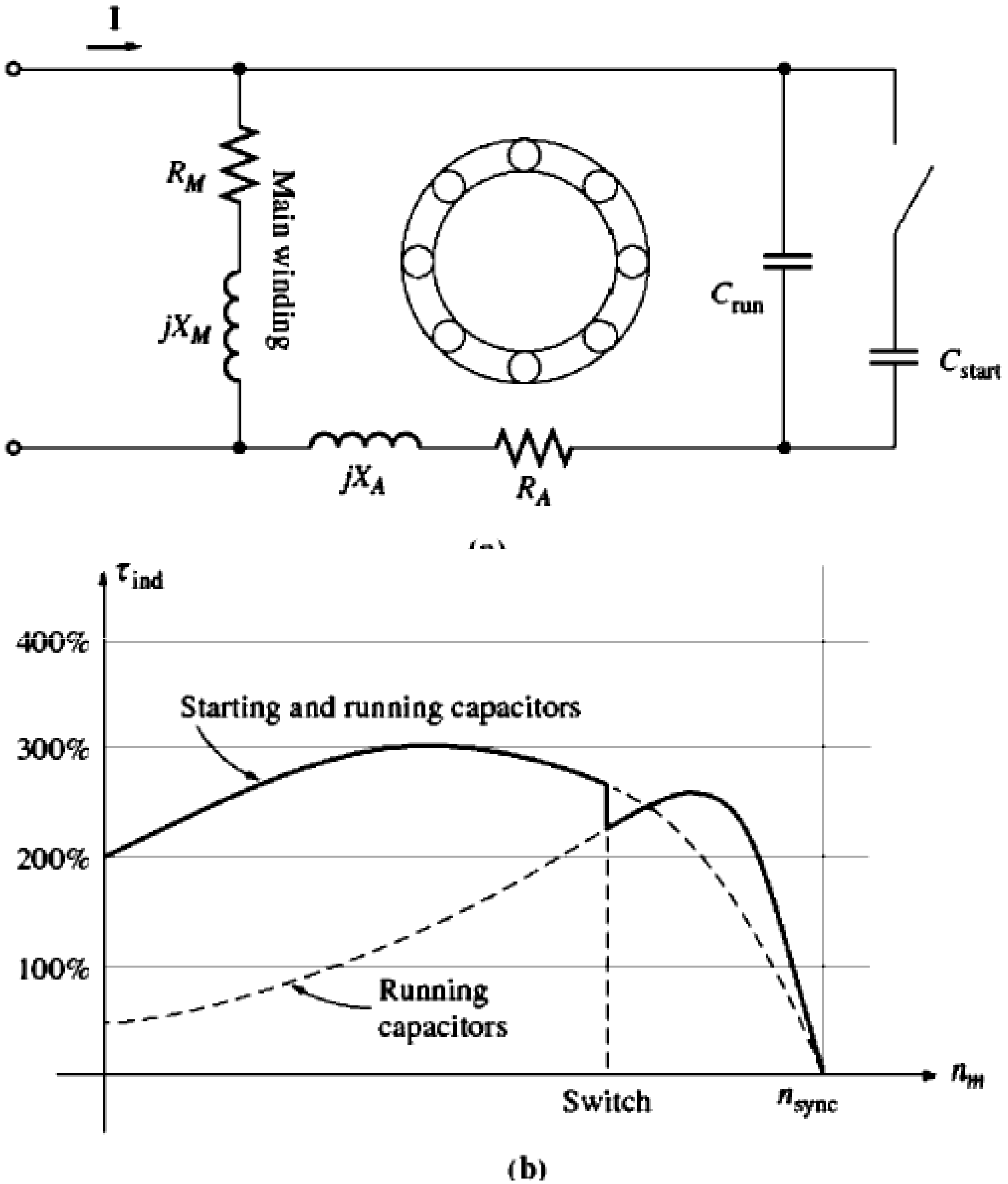


FIGURE 10–21
(a) A capacitor-start, capacitor-run induction motor. (b) Torque–speed characteristic of this motor.

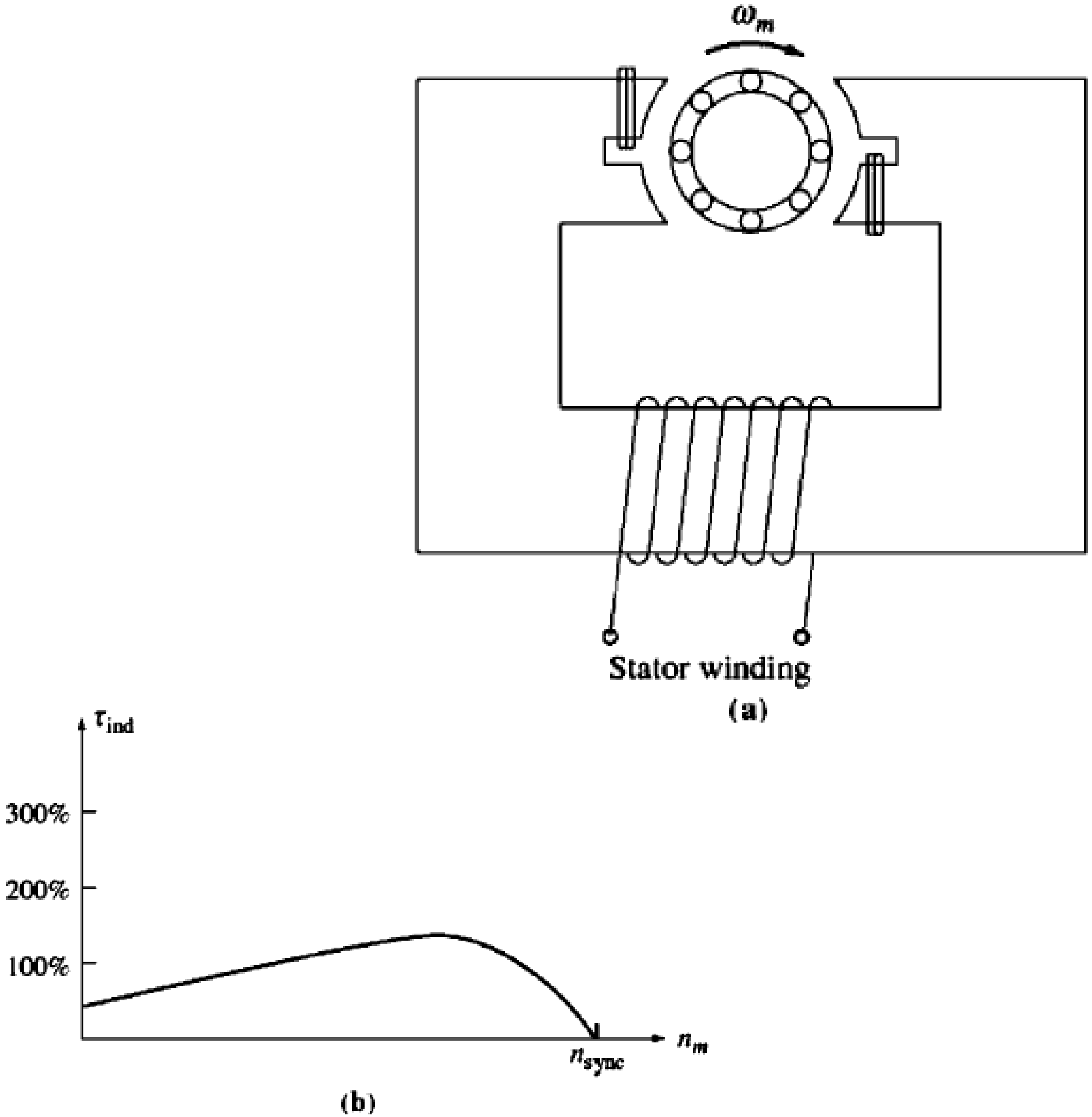


FIGURE 10–22
(a) A basic shaded-pole induction motor. (b) The resulting torque–speed characteristic.

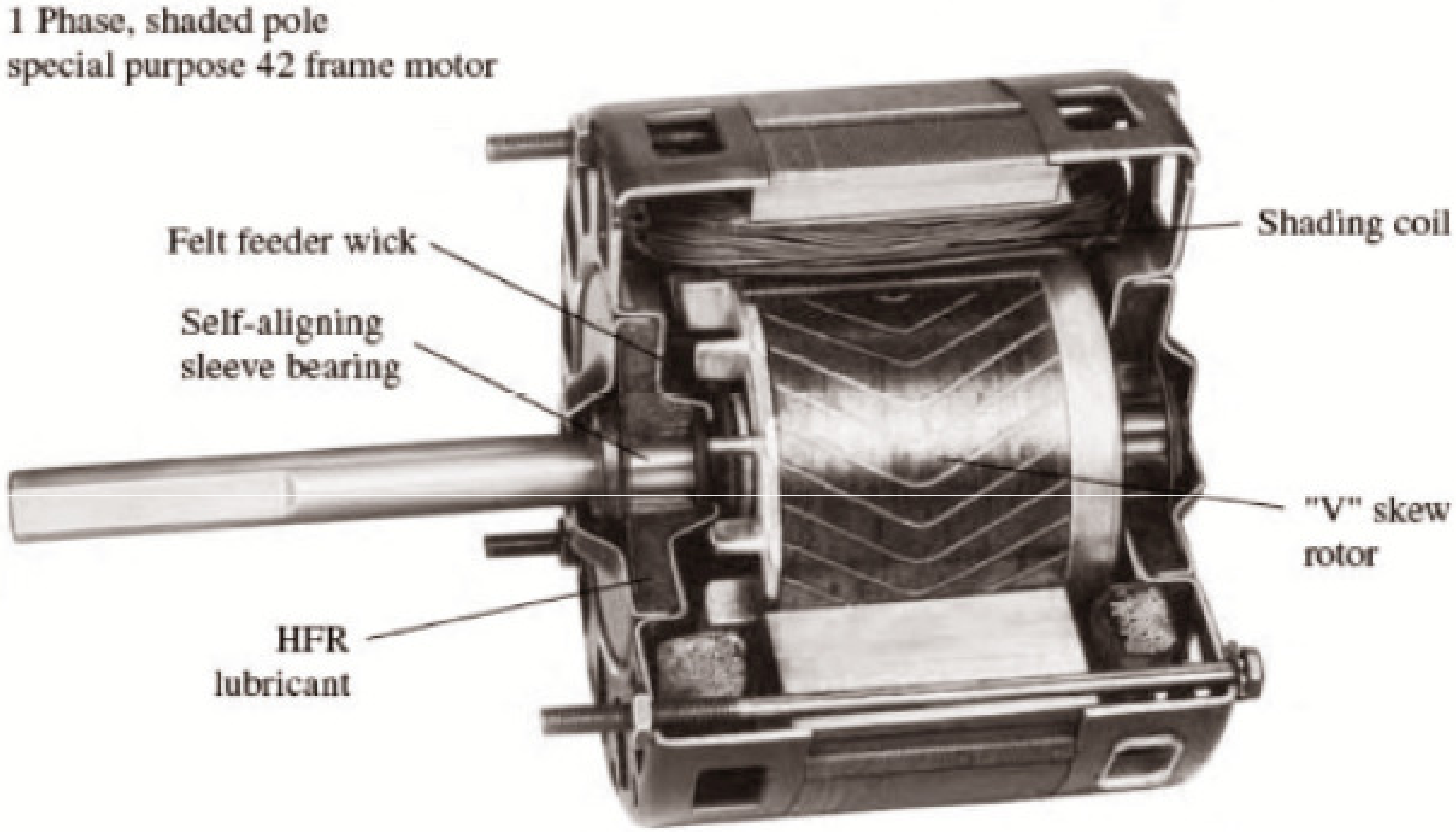


FIGURE 10–23
Cutaway view of a shaded-pole induction motor. (*Courtesy of Westinghouse Electric Corporation.*)

EXAMPLE 4.2

A four-pole synchronous ac generator with a smooth air gap has a distributed rotor winding with 263 series turns, a winding factor of 0.935, and an air gap of length 0.7 mm. Assuming the mmf drop in the electrical steel to be negligible, find the rotor-winding current required to produce a peak, space-fundamental magnetic flux density of 1.6 T in the machine air gap.

■ Solution

The space-fundamental air-gap magnetic flux density can be found by multiplying the air-gap magnetic field by the permeability of free space μ_0, which in turn can be found from the space-fundamental component of the air-gap mmf by dividing by the air-gap length g. Thus, from Eq. 4.8

$$(B_{\mathrm{ag1}})_{\mathrm{peak}} = \frac{\mu_0 (\mathcal{F}_{\mathrm{ag1}})_{\mathrm{peak}}}{g} = \frac{4\mu_0}{\pi g}\left(\frac{k_{\mathrm{r}} N_{\mathrm{r}}}{\mathrm{poles}}\right) I_{\mathrm{r}}$$

and I_r can be found from

$$I_r = \left(\frac{\pi g \cdot \text{poles}}{4\mu_0 k_r N_r}\right)(B_{\text{ag1}})_{\text{peak}}$$

$$= \left(\frac{\pi \times 0.0007 \times 4}{4 \times 4\pi \times 10^{-7} \times 0.935 \times 263}\right) 1.6$$

$$= 11.4 \text{ A}$$

EXAMPLE

A 2-pole synchronous machine has an air-gap length of 2.2 cm and a field winding with a total of 830 series turns. When excited by a field current of 47 A, the peak, space-fundamental magnetic flux density in the machine air-gap is measured to be 1.35 T.

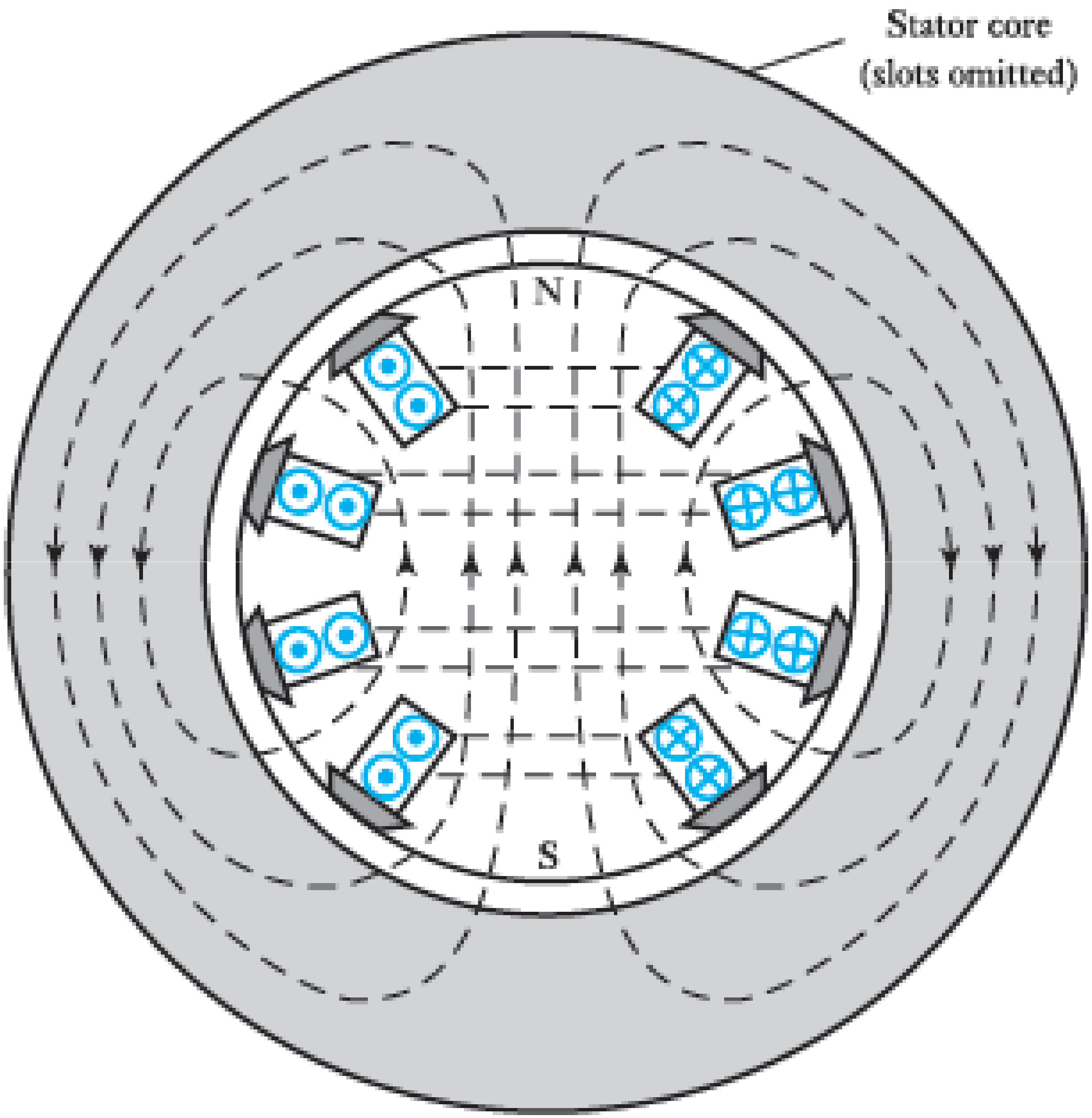
Based upon the measured flux density, calculate the field-winding winding factor k_r.

Solution

$$k_r = 0.952$$

Generated voltage

- *Field windings, in which field currents flow, are used to establish the magnetic flux required by the machine. The magnetic circuit facing this magnetic flux is so designed, and shaped such that a sinusoidal magnetic flux is produced in the air-gap.*
- *For a full pitch coil per phase placed on the inner periphery of the stator, then*


Stator core
(slots omitted)
N
S

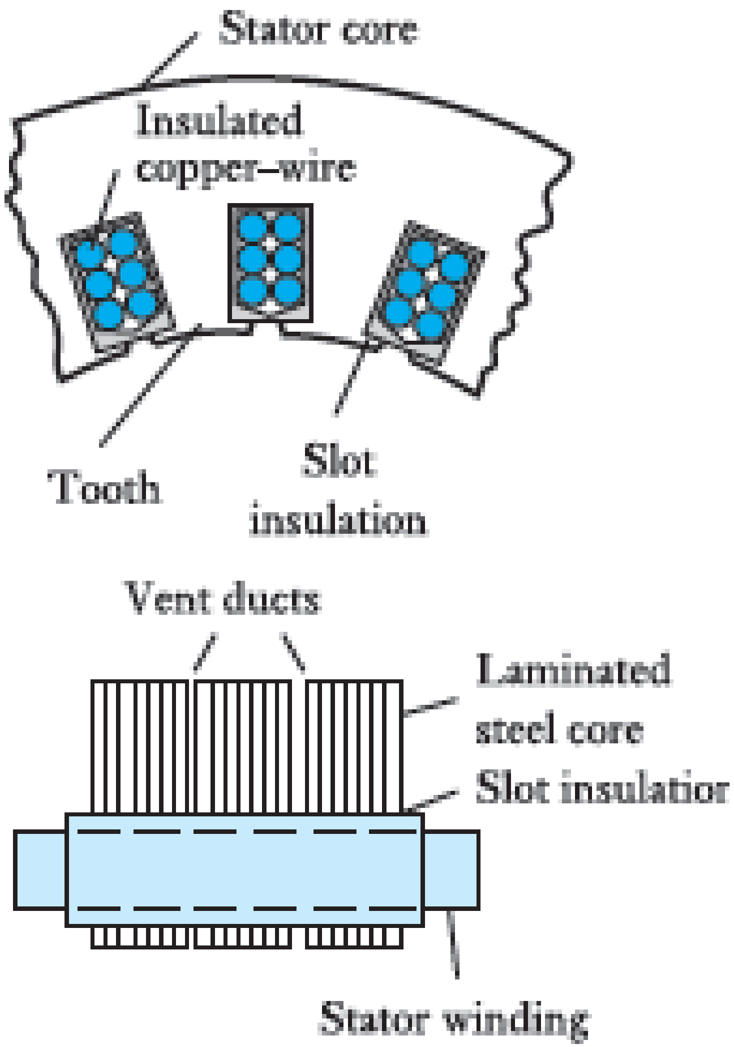


Fig. 36.1 Portion of stator of a.c. synchronous machine

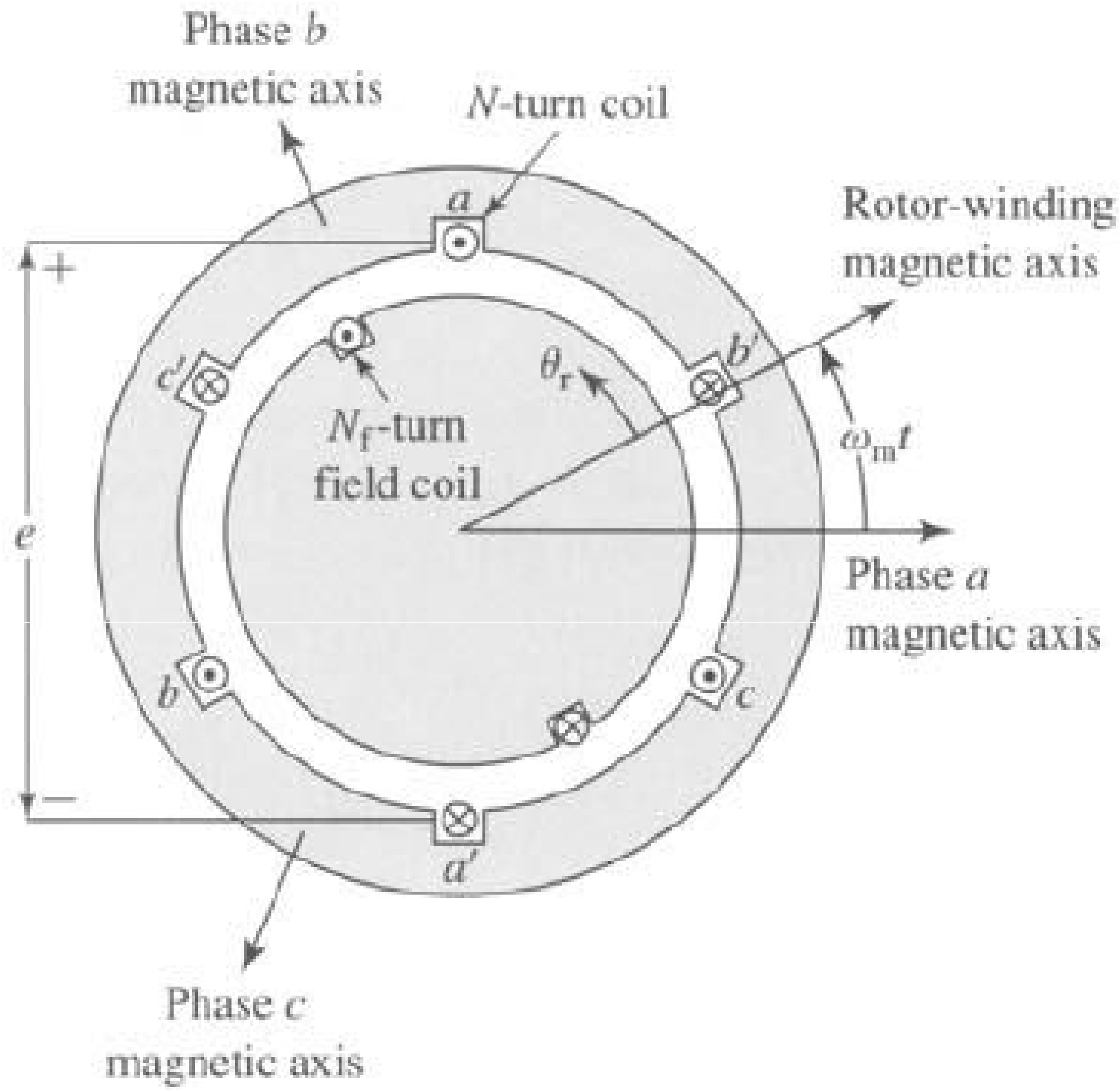


Figure 4.32 Cross-sectional view of an elementary three-phase ac machine.

- *The magnetic flux established by the field has a peak flux density along its magnetic axis given by: { next page}*
- *When the magnetic axis of the field winding coincides with the magnetic axis of the armature coils, the flux linkages of each coil reaches its maximum value as:*
- $\lambda_{max} = N_{ph} \cdot \phi_{pole}$ wb.t

Φ_{p} can be found as the integral of the flux density over the pole area

$$\Phi_{\mathrm{p}} = l \int_{-\pi/\mathrm{poles}}^{+\pi/\mathrm{poles}} B_{\mathrm{peak}} \cos\left(\frac{\mathrm{poles}}{2}\theta_{\mathrm{r}}\right) r \, d\theta_{\mathrm{r}}$$

$$= \left(\frac{2}{\mathrm{poles}}\right) 2B_{\mathrm{peak}} l r$$

Here,

θ_{r} = angle measured from the rotor magnetic axis

r = radius to air gap

l = axial length of the stator/rotor iron

$$B_{\text{peak}} = \frac{4\mu_0}{\pi g}\left(\frac{k_\text{f} N_\text{f}}{\text{poles}}\right) I_\text{f}$$

where

g = air-gap length
N_f = total series turns in the field winding
k_f = field-winding winding factor
I_f = field current

As the rotor turns, the flux linkage varies cosinusoidally with the angle between the magnetic axes of the stator coil and rotor. With the rotor spinning at constant angular velocity ω_{m}, the flux linkage with the phase-a stator coil is

$$\begin{aligned}\lambda_{\mathrm{a}} &= k_{\mathrm{w}} N_{\mathrm{ph}} \Phi_{\mathrm{p}} \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_{\mathrm{m}} t\right) \\ &= k_{\mathrm{w}} N_{\mathrm{ph}} \Phi_{\mathrm{p}} \cos \omega_{\mathrm{me}} t \qquad (4.45)\end{aligned}$$

where time t is arbitrarily chosen as zero when the peak of the flux-density wave coincides with the magnetic axis of phase a. Here,

$$\omega_{\mathrm{me}} = \left(\frac{\text{poles}}{2}\right)\omega_{\mathrm{m}} \qquad (4.46)$$

is the mechanical rotor velocity expressed in electrical rad/sec.

By Faraday's law, the voltage induced in phase *a* is

$$e_\mathrm{a} = \frac{d\lambda_\mathrm{a}}{dt} = k_\mathrm{w} N_\mathrm{ph} \frac{d\Phi_\mathrm{p}}{dt} \cos \omega_\mathrm{me} t$$
$$-\omega_\mathrm{me} k_\mathrm{w} N_\mathrm{ph} \Phi_\mathrm{p} \sin \omega_\mathrm{me} t \qquad (4.47)$$

The first term on the right-hand side of Eq. 4.47 is a transformer voltage and is present only when the amplitude of the air-gap flux wave changes with time. The second term is the *speed voltage* generated by the relative motion of the air-gap flux wave and the stator coil. In the normal steady-state operation of most rotating machines, the amplitude of the air-gap flux wave is constant; under these conditions the first term is zero and the generated voltage is simply the speed voltage. The term *electromotive force* (abbreviated *emf*) is often used for the speed voltage. Thus, for constant air-gap flux,

$$e_\mathrm{a} = -\omega_\mathrm{me} k_\mathrm{w} N_\mathrm{ph} \Phi_\mathrm{p} \sin \omega_\mathrm{me} t \qquad (4.48)$$

In the normal steady-state operation of ac machines, we are usually interested in the rms values of voltages and currents rather than their instantaneous values. From Eq. 4.48 the maximum value of the induced voltage is

$$E_{\max} = \omega_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} = 2\pi f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} \tag{4.49}$$

Its rms value is

$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} = \sqrt{2}\,\pi f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}}$$

The voltage induced in a single winding is a single-phase voltage. For the production of a set of balanced, three-phase voltages, it follows that three windings displaced 120 electrical degrees in space must be used, as shown in elementary form in Fig. 4.12. The machine of Fig. 4.12 is shown to be Y-connected and hence each winding voltage is a phase-neutral voltage. Thus, Eq. 4.50 gives the rms line-neutral voltage produced in this machine when N_{ph} is the total series turns per phase. For a Δ-connected machine, the voltage winding voltage calculated from Eq. 4.50 would be the machine line-line voltage.

EXAMPLE

A two-pole, three-phase, Y-connected 60-Hz round-rotor synchronous generator has a field winding with N_f distributed turns and winding factor k_f. The armature winding has N_a turns per phase and winding factor k_a. The air-gap length is g, and the mean air-gap radius is r. The armature-winding active length is l. The dimensions and winding data are

$N_f = 68$ series turns	$k_f = 0.945$
$N_a = 18$ series turns/phase	$k_a = 0.933$
$r = 0.53$ m	$g = 4.5$ cm
$l = 3.8$ m	

The rotor is driven by a steam turbine at a speed of 3600 r/min. For a field current of $I_f = 720$ A dc, compute (*a*) the peak fundamental mmf $(F_{ag1})_{peak}$ produced by the field winding, (*b*) the peak fundamental flux density $(B_{ag1})_{peak}$ in the air gap, (*c*) the fundamental flux per pole Φ_p, and (*d*) the rms value of the open-circuit voltage generated in the armature.

$$(F_{\mathrm{ag1}})_{\mathrm{peak}} = \frac{4}{\pi}\left(\frac{k_{\mathrm{f}}N_{\mathrm{f}}}{\mathrm{poles}}\right)I_{\mathrm{f}} = \frac{4}{\pi}\left(\frac{0.945 \times 68}{2}\right)720$$

$$= \frac{4}{\pi}(32.1)720 = 2.94 \times 10^4\ \mathrm{A \cdot turns/pole}$$

$$(B_{\mathrm{ag1}})_{\mathrm{peak}} = \frac{\mu_0(F_{\mathrm{ag1}})_{\mathrm{peak}}}{g} = \frac{4\pi \times 10^{-7} \times 2.94 \times 10^4}{4.5 \times 10^{-2}} = 0.821\ \mathrm{T}$$

$$\Phi_{\mathrm{p}} = 2(B_{\mathrm{ag1}})_{\mathrm{peak}}lr = 2(0.821)(3.8)(0.53) = 3.31\ \mathrm{Wb}$$

$$E_{\mathrm{rms,line-neutral}} = \sqrt{2}\,\pi f_{\mathrm{me}}k_{\mathrm{a}}N_{\mathrm{a}}\Phi_{\mathrm{p}} = \sqrt{2}\,\pi(60)(0.933)(18)(3.31)$$

$$= 14.8\,\mathrm{kV\ rms}$$

The line-line voltage is thus

$$E_{\mathrm{rms,line-line}} = \sqrt{3}\,(14.8\,\mathrm{kV}) = 25.7\,\mathrm{kV\ rms}$$

Generated voltage in dc machines

- *In dc machines the emf induced in armature winding is sinusoidal one. Then by using mechanical converter known as the comutator, the ac induced emf is rectified to dc one.*
- *By using large number of armature conductors, the total induced voltage is increased. Its average value is found as:*

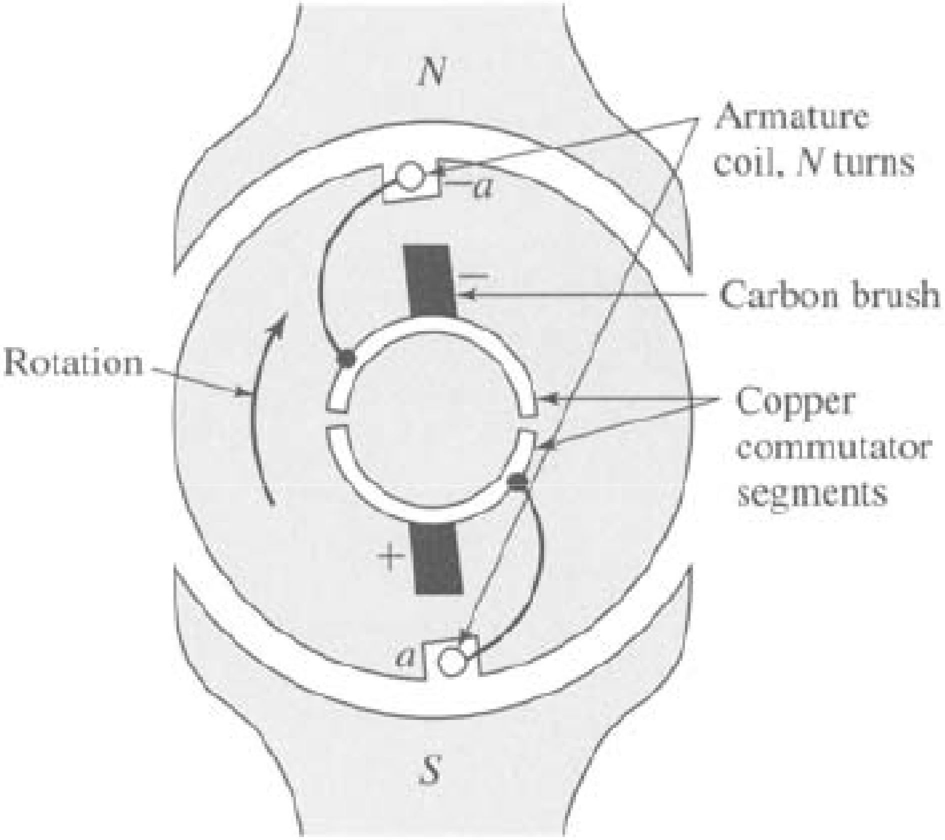


Figure 4.17 Elementary dc machine with commutator.

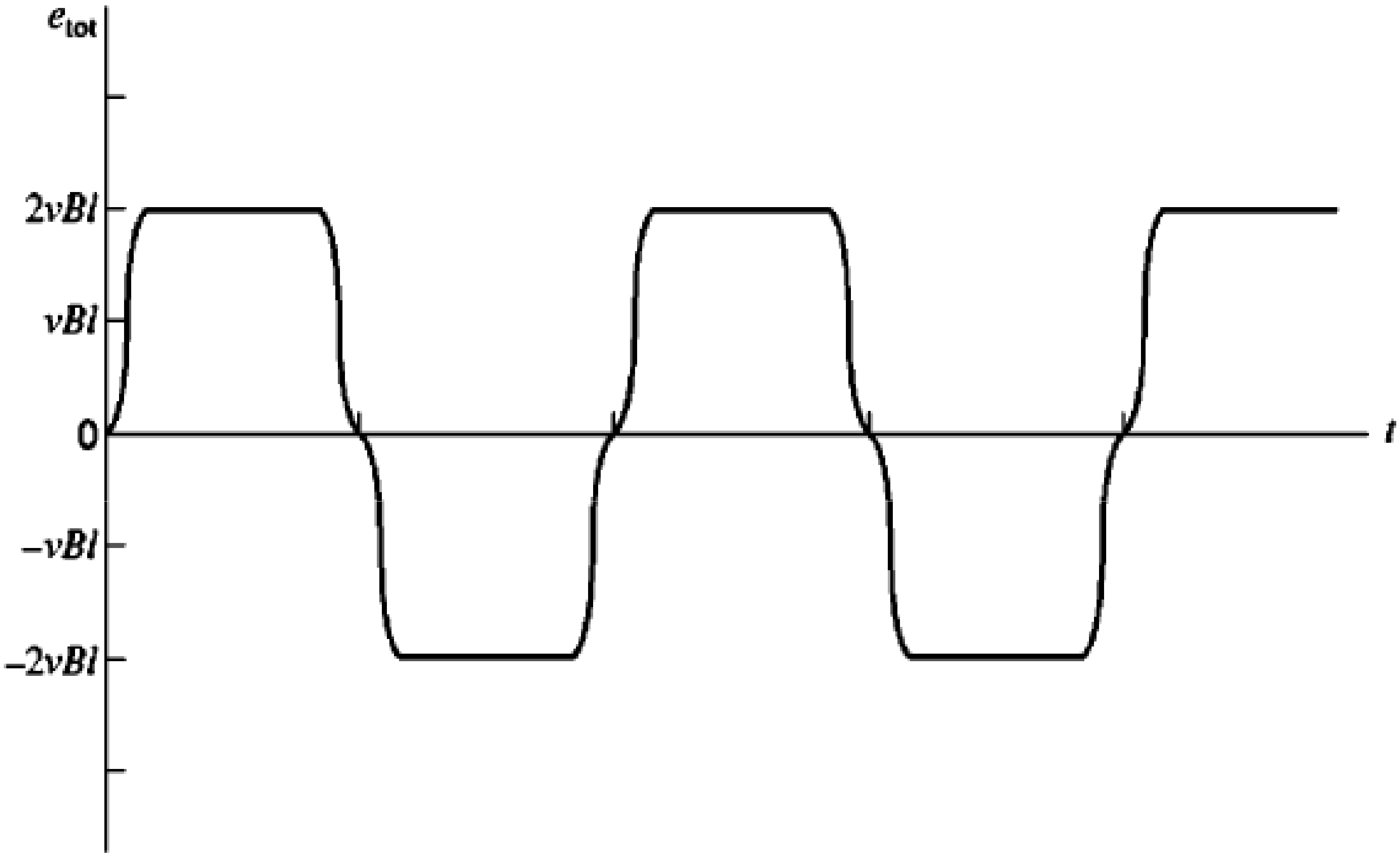
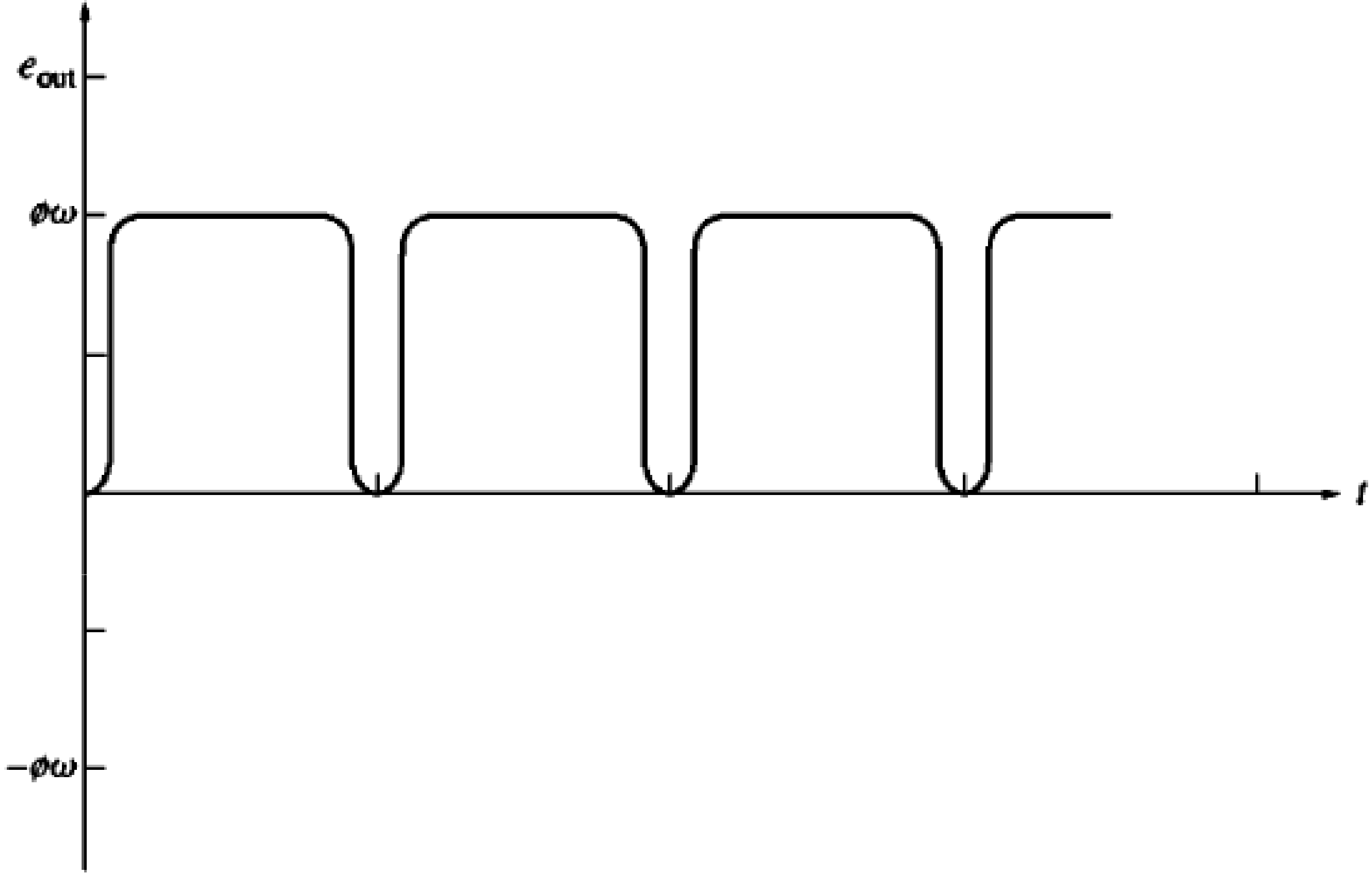


FIGURE 8–3
The output voltage of the loop.



(b)

FIGURE 8–5
Producing a dc output from the machine with a commutator and brushes. (a) Perspective view; (b) the resulting output voltage.

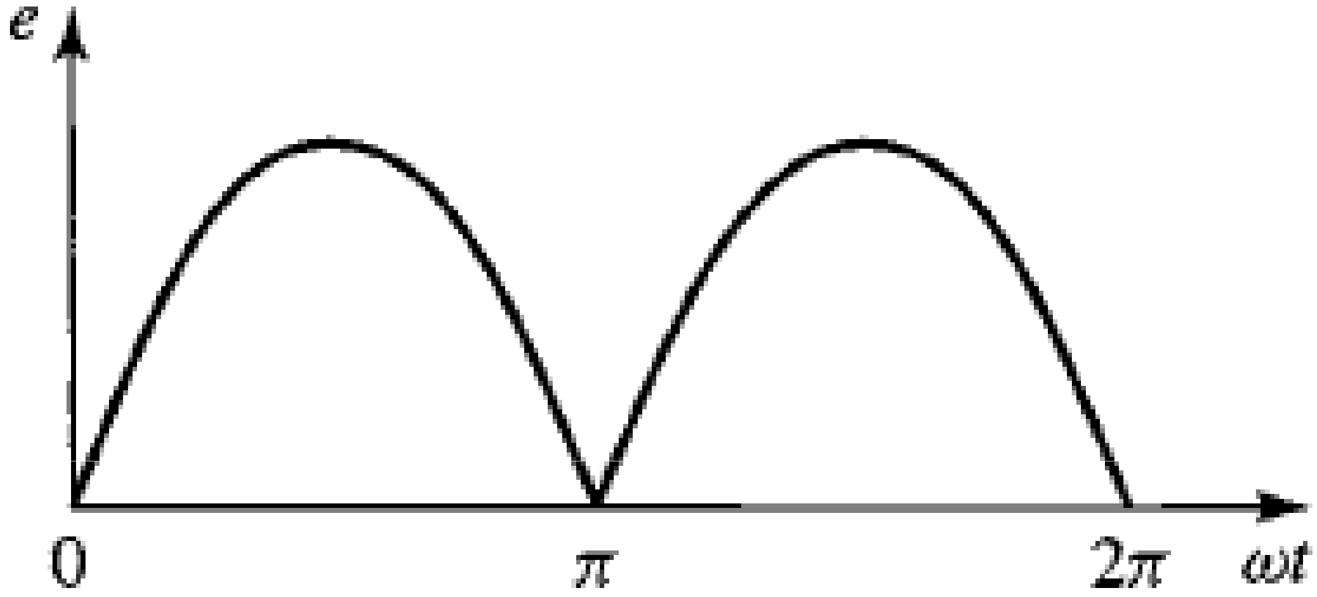


Figure 4.33 Voltage between the brushes in the elementary dc machine of Fig. 4.17.

$$E_{\mathrm{a}} = \frac{1}{\pi}\int_0^{\pi} \omega_{\mathrm{me}} N \Phi_{\mathrm{p}} \sin(\omega_{\mathrm{me}} t)\, d(\omega_{\mathrm{me}} t) = \frac{2}{\pi}\omega_{\mathrm{me}} N \Phi_{\mathrm{p}}$$

$$\omega_{\mathrm{me}} = \left(\frac{\text{poles}}{2}\right)\omega_{\mathrm{m}} \qquad N_{\mathrm{a}} = C_{\mathrm{a}}/(2m)$$

$$E_{\mathrm{a}} = \left(\frac{\text{poles}}{\pi}\right) N \Phi_{\mathrm{p}} \omega_{\mathrm{m}} = \text{poles}\, N \Phi_{\mathrm{p}} \left(\frac{n}{30}\right)$$

$$E_{\mathrm{a}} = \left(\frac{\text{poles}}{2\pi}\right)\left(\frac{C_{\mathrm{a}}}{m}\right)\Phi_{\mathrm{p}}\omega_{\mathrm{m}} = \left(\frac{\text{poles}}{60}\right)\left(\frac{C_{\mathrm{a}}}{m}\right)\Phi_{\mathrm{p}}\, n$$

Developed torque

- *Case 1- Developed torque in non-salient machines:*
- *Generally in all electric machines, the stator windings will establish a resultant stator magnetic field {stationary or rotating one}. This field will appear as a magnet with north and south poles on the inner side of the stator.*
- *On the other hand rotor windings will produce their own magnetic field {stationary or rotating one}. It will also appear as north and south poles,*

- *On the outer side of the rotor.*
- *Torque is produced by the tendency of the stator and rotor magnetic field to align.*
- *Continuous torque is developed as long as these two field are displaced from each other by certain angle, and stationary with respect to each other.*

First: Coupled circuit point of view:

- *For machines with uniform air gaps, stator and rotor self inductances are constant independent of rotor position. The only inductances that are variable with rotor position are the mutual inductances between rotor and stator windings.*

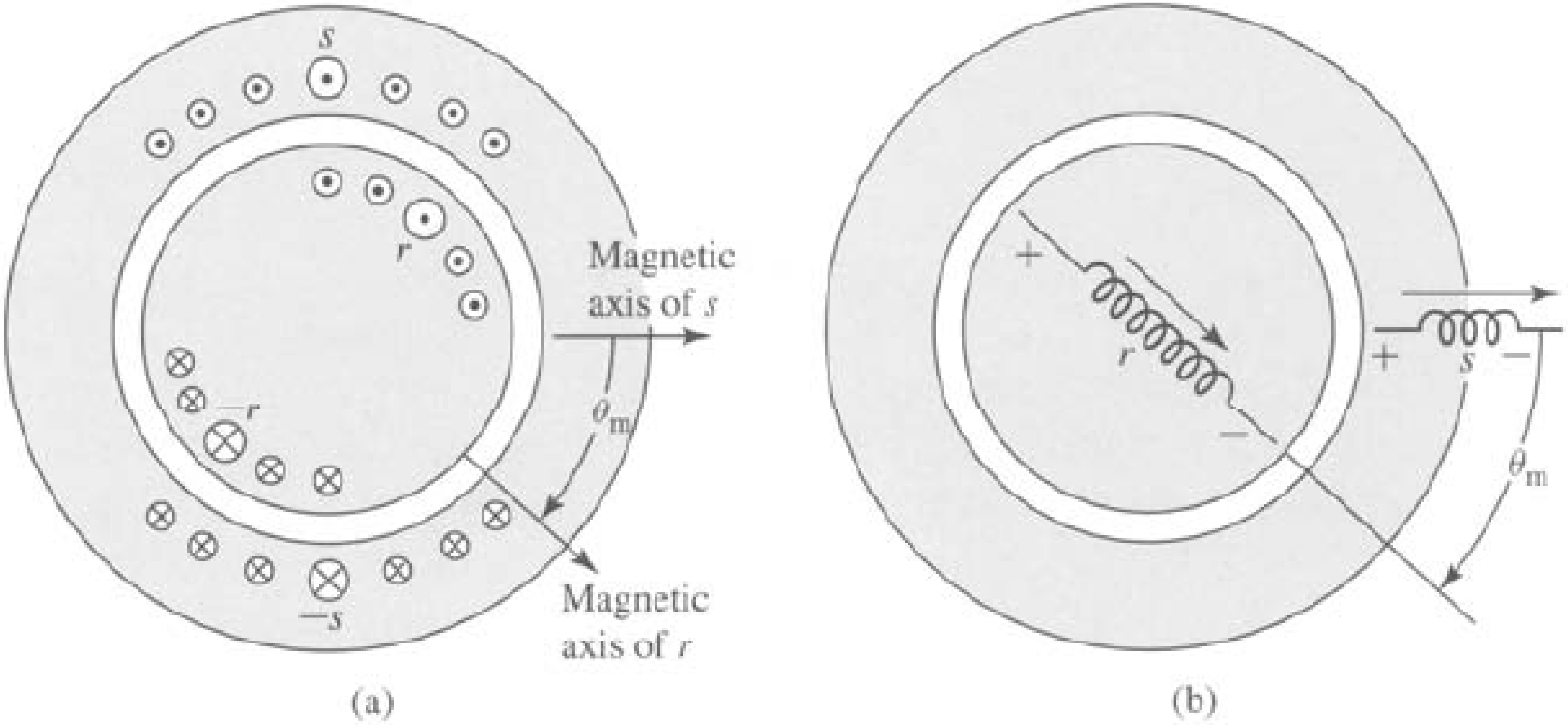


Figure 4.34 Elementary two-pole machine with smooth air gap: (a) winding distribution and (b) schematic representation.

$$\mathcal{L}_{\mathrm{sr}}(\theta_{\mathrm{me}}) = L_{\mathrm{sr}} \cos{(\theta_{\mathrm{me}})}$$

$$\lambda_{\mathrm{s}} = L_{\mathrm{ss}} i_{\mathrm{s}} + \mathcal{L}_{\mathrm{sr}}(\theta_{\mathrm{me}}) i_{\mathrm{r}} = L_{\mathrm{ss}} i_{\mathrm{s}} + L_{\mathrm{sr}} \cos{(\theta_{\mathrm{me}})} i_{\mathrm{r}}$$

$$\lambda_{\mathrm{r}} = \mathcal{L}_{\mathrm{sr}}(\theta_{\mathrm{me}}) i_{\mathrm{s}} + L_{\mathrm{rr}} i_{\mathrm{r}} = L_{\mathrm{sr}} \cos{(\theta_{\mathrm{me}})} i_{\mathrm{s}} + L_{\mathrm{rr}} i_{\mathrm{r}}$$

$$\begin{bmatrix} \lambda_{\mathrm{s}} \\ \lambda_{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} L_{\mathrm{ss}} & \mathcal{L}_{\mathrm{sr}}(\theta_{\mathrm{me}}) \\ \mathcal{L}_{\mathrm{sr}}(\theta_{\mathrm{me}}) & L_{\mathrm{rr}} \end{bmatrix} \begin{bmatrix} i_{\mathrm{s}} \\ i_{\mathrm{r}} \end{bmatrix}$$

The terminal voltages v_{s} and v_{r} are

$$v_{\mathrm{s}} = R_{\mathrm{s}} i_{\mathrm{s}} + \frac{d\lambda_{\mathrm{s}}}{dt}$$

$$v_{\mathrm{r}} = R_{\mathrm{r}} i_{\mathrm{r}} + \frac{d\lambda_{\mathrm{r}}}{dt}$$

$$v_{\mathrm{s}} = R_{\mathrm{s}} i_{\mathrm{s}} + L_{\mathrm{ss}} \frac{di_{\mathrm{s}}}{dt} + L_{\mathrm{sr}} \cos(\theta_{\mathrm{me}}) \frac{di_{\mathrm{r}}}{dt} - L_{\mathrm{sr}} i_{\mathrm{r}} \sin(\theta_{\mathrm{me}}) \frac{d\theta_{\mathrm{me}}}{dt}$$

$$v_{\mathrm{r}} = R_{\mathrm{r}} i_{\mathrm{r}} + L_{\mathrm{rr}} \frac{di_{\mathrm{r}}}{dt} + L_{\mathrm{sr}} \cos(\theta_{\mathrm{me}}) \frac{di_{\mathrm{r}}}{dt} - L_{\mathrm{sr}} i_{\mathrm{s}} \sin(\theta_{\mathrm{me}}) \frac{d\theta_{\mathrm{me}}}{dt}$$

$$\frac{d\theta_{\mathrm{me}}}{dt} = \omega_{\mathrm{me}} = \left(\frac{\mathrm{poles}}{2}\right)\omega_{\mathrm{m}}$$

$$\begin{aligned} W'_{\mathrm{fld}} &= \frac{1}{2}L_{\mathrm{ss}}i_{\mathrm{s}}^2 + \frac{1}{2}L_{\mathrm{rr}}i_{\mathrm{r}}^2 + L_{\mathrm{sr}}i_{\mathrm{s}}i_{\mathrm{r}}\cos\theta_{\mathrm{me}} \\ &= \frac{1}{2}L_{\mathrm{ss}}i_{\mathrm{s}}^2 + \frac{1}{2}L_{\mathrm{rr}}i_{\mathrm{r}}^2 + L_{\mathrm{sr}}i_{\mathrm{s}}i_{\mathrm{r}}\cos\left(\left(\frac{\mathrm{poles}}{2}\right)\theta_{\mathrm{m}}\right) \end{aligned}$$

$$T = \left.\frac{\partial W'_{\text{fld}}(i_{\text{s}}, i_{\text{r}}, \theta_{\text{m}})}{\partial \theta_{\text{m}}}\right|_{i_{\text{s}}, i_{\text{r}}} = -\left(\frac{\text{poles}}{2}\right) L_{\text{sr}} i_{\text{s}} i_{\text{r}} \sin\left(\frac{\text{poles}}{2}\theta_{\text{m}}\right)$$

$$= -\left(\frac{\text{poles}}{2}\right) L_{\text{sr}} i_{\text{s}} i_{\text{r}} \sin\theta_{\text{me}}$$

where T is the electromechanical torque acting to accelerate the rotor (i.e., a positive torque acts to increase θ_{m}). The negative sign in Eq. 4.65 means that the electromechanical torque acts in the direction to bring the magnetic fields of the stator and rotor into alignment.

EXAMPLE

Consider the elementary two-pole, two-winding machine of Fig. 4.34. Its shaft is coupled to a mechanical device which can be made to absorb or deliver mechanical torque over a wide range of speeds. This machine can be connected and operated in several ways. For this example, let us consider the situation in which the rotor winding is excited with direct current I_r and the stator winding is connected to an ac source which can either absorb or deliver electric power. Let the stator current be

$$i_s = I_s \cos \omega_e t$$

where $t = 0$ is arbitrarily chosen as the moment when the stator current has its peak value.

a. Derive an expression for the magnetic torque developed by the machine as the speed is varied by control of the mechanical device connected to its shaft.
b. Find the speed at which average torque will be produced if the stator frequency is 60 Hz.
c. With the assumed current-source excitations, what voltages are induced in the stator and rotor windings at synchronous speed ($\omega_m = \omega_e$)?

Solution

a. From Eq. 4.65 for a two-pole machine

$$T = -L_{sr} i_s i_r \sin \theta_m$$

For the conditions of this problem, with $\theta_m = \omega_m t + \delta$

$$T = -L_{sr} I_s I_r \cos \omega_e t \sin (\omega_m t + \delta)$$

where ω_m is the clockwise angular velocity impressed on the rotor by the mechanical drive and δ is the angular position of the rotor at $t = 0$. Using a trigonometric identity,[2] we have

$$T = -\frac{1}{2} L_{sr} I_s I_r \{\sin [(\omega_m + \omega_e)t + \delta] + \sin [(\omega_m - \omega_e)t + \delta]\}$$

The torque consists of two sinusoidally time-varying terms of frequencies $\omega_m + \omega_e$ and $\omega_m - \omega_e$. As shown in Section 4.5, ac current applied to the two-pole, single-phase stator winding in the machine of Fig. 4.34 creates two flux waves, one traveling in the positive θ_m direction with angular velocity ω_e and the second traveling in the negative θ_m direction also with angular velocity ω_e. It is the interaction of the rotor with these two flux waves which results in the two components of the torque expression.

b. Except when $\omega_m = \pm\omega_e$, the torque averaged over a sufficiently long time is zero. But if $\omega_m = \omega_e$, the rotor is traveling in synchronism with the positive-traveling stator flux wave, and the torque becomes

$$T = -\frac{1}{2} L_{sr} I_s I_r [\sin(2\omega_e t + \delta) + \sin\delta]$$

The first sine term is a double-frequency component whose average value is zero. The second term is the average torque

$$T_{\mathrm{avg}} = -\frac{1}{2} L_{\mathrm{sr}} I_{\mathrm{s}} I_{\mathrm{r}} \sin \delta$$

A nonzero average torque will also be produced when $\omega_{\mathrm{m}} = -\omega_{\mathrm{e}}$ which merely means rotation in the counterclockwise direction; the rotor is now traveling in synchronism with the negative-traveling stator flux wave. The negative sign in the expression for T_{avg} means that a positive value of T_{avg} acts to reduce δ.

c. From the second and fourth terms of Eq. 4.61 (with $\theta_{\mathrm{e}} = \theta_{\mathrm{m}} = \omega_{\mathrm{m}} t + \delta$), the voltage induced in the stator when $\omega_{\mathrm{m}} = \omega_{\mathrm{e}}$ is

$$e_{\mathrm{s}} = -\omega_{\mathrm{e}} L_{\mathrm{ss}} I_{\mathrm{s}} \sin \omega_{\mathrm{e}} t - \omega_{\mathrm{e}} L_{\mathrm{sr}} I_{\mathrm{r}} \sin (\omega_{\mathrm{e}} t + \delta)$$

$$\begin{aligned} e_{\mathrm{r}} &= -\omega_0 L_{\mathrm{sr}} I_{\mathrm{s}} [\sin \omega_{\mathrm{e}} t \cos (\omega_{\mathrm{e}} t + \delta) + \cos \omega_{\mathrm{s}} t \sin (\omega_{\mathrm{e}} t + \delta)] \\ &= -\omega_{\mathrm{e}} L_{\mathrm{sr}} I_{\mathrm{s}} \sin (2\omega_{\mathrm{e}} t + \delta) \end{aligned}$$

The backwards-rotating component of the stator flux induces a double-frequency voltage in the rotor, while the forward-rotating component, which is rotating in sychronism with the rotor, appears as a dc flux to the rotor, and hence induces no voltage in the rotor winding.

EXAMPLE

Consider a 4-pole, three-phase synchronous machine with a uniform air gap. Assume the armature-winding self- and mutual inductances to be constant

$$L_{\mathrm{aa}} = L_{\mathrm{bb}} = L_{\mathrm{cc}}$$

$$L_{\mathrm{ab}} = L_{\mathrm{bc}} = L_{\mathrm{ca}}$$

$$\mathcal{L}_{\mathrm{af}} = L_{\mathrm{af}} \cos 2\theta_{\mathrm{m}}$$

$$\mathcal{L}_{\mathrm{bf}} = L_{\mathrm{af}} \cos (2\theta_{\mathrm{m}} - 120^\circ)$$

$$\mathcal{L}_{\mathrm{cf}} = L_{\mathrm{af}} \cos (2\theta_{\mathrm{m}} + 120^\circ)$$

assume the field-winding self-inductance L_{f} to be constant

Show that when the field is excited with constant current I_f and the armature is excited by balanced-three-phase currents of the form

$$i_a = I_a \cos(\omega_e t + \delta)$$

$$i_b = I_a \cos(\omega_e t - 120^\circ + \delta)$$

$$i_c = I_a \cos(\omega_e t + 120^\circ + \delta)$$

the torque will be constant when the rotor travels at synchronous speed ω_s as given by Eq. 4.40.

Solution

$$\begin{aligned}
W'_{\text{fld}}(i_a, i_b, i_c, i_f, \theta_m) &= (\text{constant terms}) + \mathcal{L}_{af} i_a i_f + \mathcal{L}_{bf} i_b i_f + \mathcal{L}_{cf} i_c i_f \\
&= (\text{constant terms}) + L_{af} I_a I_f [\cos 2\theta_m \cos(\omega_e t + \delta) \\
&\quad + \cos(2\theta_m - 120^\circ)\cos(\omega_e t - 120^\circ + \delta) \\
&\quad + \cos(2\theta_m + 120^\circ)\cos(\omega_e t + 120^\circ + \delta)] \\
&= (\text{constant terms}) + \frac{3}{2} L_{af} I_a I_f \cos(2\theta_m - \omega_e t - \delta)
\end{aligned}$$

$$T = \left.\frac{\partial W'_{\mathrm{fld}}}{\partial \theta_{\mathrm{m}}}\right|_{i_{\mathrm{a}}, i_{\mathrm{b}}, i_{\mathrm{c}}, i_{\mathrm{f}}}$$

$$= -3 L_{\mathrm{af}} I_{\mathrm{a}} I_{\mathrm{f}} \sin\left(2\theta_{\mathrm{m}} - \omega_{\mathrm{e}} t - \delta\right)$$

$$\theta_{\mathrm{m}} = \omega_{\mathrm{s}} t = \left(\frac{\omega_{\mathrm{e}}}{2}\right) t$$

$$T = 3 L_{\mathrm{af}} I_{\mathrm{a}} I_{\mathrm{f}} \sin \delta$$

Note that unlike the case of the single-phase machine of Example 4.6, the torque for this three-phase machine operating at synchronous velocity under balanced-three-phase conditions is constant. As we have seen, this is due to the fact that the stator mmf wave consists of a single rotating flux wave, as opposed to the single-phase case in which the stator phase current produces both a forward- and a backward-rotating flux wave. This backwards flux wave is not in synchronism with the rotor and hence is responsible for the double-frequency time-varying torque component seen in Example 4.6.

EXAMPLE

For the four-pole machine of Example 4.7, find the synchronous speed at which a constant torque will be produced if the rotor currents are of the form

$$i_{\mathrm{a}} = I_{\mathrm{a}} \cos\left(\omega_{\mathrm{e}} t + \delta\right)$$

$$i_{\mathrm{b}} = I_{\mathrm{a}} \cos\left(\omega_{\mathrm{e}} t + 120^\circ + \delta\right)$$

$$i_{\mathrm{c}} = I_{\mathrm{a}} \cos\left(\omega_{\mathrm{e}} t - 120^\circ + \delta\right)$$

Solution

half of the electrical excitation frequency. This result can be generalized to show that, under balanced operating conditions, a multiphase, multipole synchronous machine will produce constant torque at a rotor speed, at which the rotor rotates in synchronism with the rotating flux wave produced by the stator currents. Hence, this is known as the *synchronous speed* of the machine. From Eqs. 4.40 and 4.41, the synchronous speed is equal to $\omega_s = (2/\text{poles})\omega_e$ in rad/sec or $n_s = (120/\text{poles}) f_e$ in r/min.

$$\omega_s = -(\omega_e/2)$$

Magnetic field view point

- *Here the stator and rotor windings will establish magnetic flux in the machine, with each one appear as a permanent pole centered along the magnetic axis of that winding { they could be stationary or rotating, but the must be relatively stationary with respect to each other}. See figure shown below*

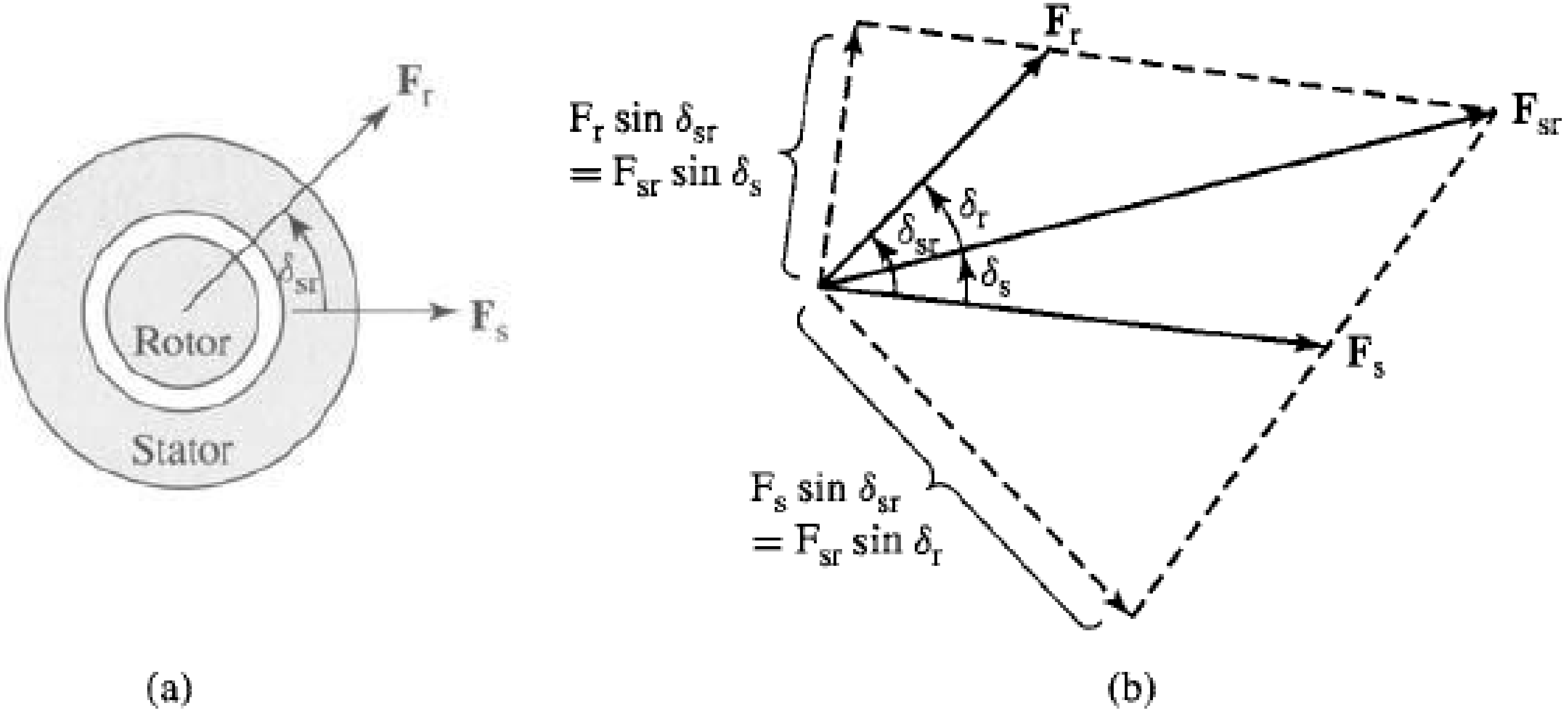


Figure 4.35 Simplified two-pole machine: (a) elementary model and (b) vector diagram of mmf waves. Torque is produced by the tendency of the rotor and stator magnetic fields to align. Note that these figures are drawn with δ_{sr} positive, i.e., with the rotor mmf wave $\mathbf{F}_r$ leading that of the stator $\mathbf{F}_s$.

Only the mutual flux is of direct concern in torque production.

$$H_{\mathrm{ag}}g = \mathcal{F}_{\mathrm{sr}}$$

where the script $\mathcal{F}$ denotes the mmf wave as a function of the angle around the periphery.

$$F_{\mathrm{sr}}^2 = F_{\mathrm{s}}^2 + F_{\mathrm{r}}^2 + 2F_{\mathrm{s}}F_{\mathrm{r}}\cos\delta_{\mathrm{sr}}$$

$$(H_{\mathrm{ag}})_{\mathrm{peak}} = \frac{F_{\mathrm{sr}}}{g}$$

$$\text{Average coenergy density} = \frac{\mu_0}{2}\left(\frac{(H_{\mathrm{ag}})^2_{\mathrm{peak}}}{2}\right) = \frac{\mu_0}{4}\left(\frac{F_{\mathrm{sr}}}{g}\right)^2$$

$$W'_{\text{fld}} = (\text{average coenergy density})(\text{volume of air gap})$$

$$= \frac{\mu_0}{4}\left(\frac{F_{\text{sr}}}{g}\right)^2 \pi D l g = \frac{\mu_0 \pi D l}{4g} F_{\text{sr}}^2$$

where l is the axial length of the air gap and D is its average diameter.

$$W'_{\text{fld}} = \frac{\mu_0 \pi D l}{4g}\left(F_{\text{s}}^2 + F_{\text{r}}^2 + 2F_{\text{s}}F_{\text{r}}\cos\delta_{\text{sr}}\right)$$

Recognizing that holding mmf constant is equivalent to holding current constant, an expression for the electromechanical torque T can now be obtained in terms of the

For a two-pole machine

$$T = \left.\frac{\partial W'_{\text{fld}}}{\partial \delta_{\text{sr}}}\right|_{F_{\text{s}}, F_{\text{r}}} = -\left(\frac{\mu_0 \pi D l}{2g}\right) F_{\text{s}} F_{\text{r}} \sin\delta_{\text{sr}}$$

The general expression for the torque for a multipole machine is

$$T = -\left(\frac{\text{poles}}{2}\right)\left(\frac{\mu_0 \pi D l}{2g}\right) F_\mathrm{s} F_\mathrm{r} \sin \delta_\mathrm{sr}$$

In this equation, δ_sr is the electrical space-phase angle between the rotor and stator mmf waves and the torque T acts in the direction to accelerate the rotor. Thus when δ_sr is positive, the torque is negative and the machine is operating as a generator. Similarly, a negative value of δ_sr corresponds to positive torque and, correspondingly, motor action.

This important equation states that the torque is proportional to the peak values of the stator- and rotor-mmf waves F_s and F_r and to the sine of the electrical space-phase angle δ_sr between them. The minus sign means that the fields tend to align themselves. Equal and opposite torques are exerted on the stator and rotor. The torque on the stator is transmitted through the frame of the machine to the foundation.

Also note that in Fig. 4.35b

$$F_s \sin \delta_{sr} = F_{sr} \sin \delta_r$$

$$F_r \sin \delta_{sr} = F_{sr} \sin \delta_s$$

$$T = -\left(\frac{\text{poles}}{2}\right)\left(\frac{\mu_0 \pi D l}{2g}\right) F_s F_{sr} \sin \delta_s$$

$$T = -\left(\frac{\text{poles}}{2}\right)\left(\frac{\mu_0 \pi D l}{2g}\right) F_r F_{sr} \sin \delta_r$$

In Eqs. 4.73, 4.76, and 4.77, the fields have been expressed in terms of the peak values of their mmf waves. When magnetic saturation is neglected, the fields can, of course, be expressed in terms of the peak values of their flux-density waves or in terms of total flux per pole. Thus the peak value B_{ag} of the field due to a sinusoidally distributed mmf wave in a uniform-air-gap machine is $\mu_0 F_{\mathrm{ag,peak}}/g$, where $F_{\mathrm{ag,peak}}$ is the peak value of the mmf wave. For example, the resultant mmf F_{sr} produces a resultant flux-density wave whose peak value is $B_{\mathrm{sr}} = \mu_0 F_{\mathrm{sr}}/g$. Thus, $F_{\mathrm{sr}} = g B_{\mathrm{sr}}/\mu_0$

$$T = -\left(\frac{\text{poles}}{2}\right)\left(\frac{\pi D l}{2}\right) B_{\mathrm{sr}} F_{\mathrm{r}} \sin \delta_{\mathrm{r}}$$

EXAMPLE

An 1800-r/min, four-pole, 60-Hz synchronous motor has an air-gap length of 1.2 mm. The average diameter of the air-gap is 27 cm, and its axial length is 32 cm. The rotor winding has 786 turns and a winding factor of 0.976. Assuming that thermal considerations limit the rotor current to 18 A, estimate the maximum torque and power output one can expect to obtain from this machine.

$$(F_r)_{\max} = \frac{4}{\pi}\left(\frac{k_r N_r}{\text{poles}}\right)(I_r)_{\max} = \frac{4}{\pi}\left(\frac{0.976 \times 786}{4}\right) 18 = 4395\ \text{A}$$

Assuming that the peak value of the resultant air-gap flux is limited to 1.5 T,

$$\begin{aligned} T_{\max} &= \left(\frac{\text{poles}}{2}\right)\left(\frac{\pi D l}{2}\right) B_{\text{sr}} (F_r)_{\max} \\ &= \left(\frac{4}{2}\right)\left(\frac{\pi \times 0.27 \times 0.32}{2}\right) 1.5 \times 4400 = 1790\ \text{N} \cdot \text{m} \end{aligned}$$

For a synchronous speed of 1800 r/min, $\omega_m = n_s\,(\pi/30) = 1800\,(\pi/30) = 60\pi$ rad/sec, and thus the corresponding power can be calculated as $P_{\max} = \omega_m T_{\max} = 337$ kW.

Alternative forms of the torque equation arise when it is recognized that

$$\Phi_p = (\text{average value of B over a pole})(\text{pole area})$$

$$\Phi_p = \frac{2}{\pi} B_{\text{peak}} \left(\frac{\pi D l}{\text{poles}} \right) = \left(\frac{2 D l}{\text{poles}} \right) B_{\text{peak}}$$

$$T = -\frac{\pi}{2} \left(\frac{\text{poles}}{2} \right)^2 \Phi_{sr} F_r \sin \delta_r$$

where Φ_{sr} is the resultant flux per pole produced by the combined effect of the stator and rotor mmf's.

To recapitulate, we now have several forms in which the torque of a uniform-air-gap machine can be expressed in terms of its magnetic fields. *All are merely statements that the torque is proportional to the product of the magnitudes of the interacting fields and to the sine of the electrical space angle between their magnetic axes.* The negative sign indicates that the electromechanical torque acts in a direction to decrease the displacement angle between the fields. In our preliminary discussion of machine types, Eq. 4.81 will be the preferred form.